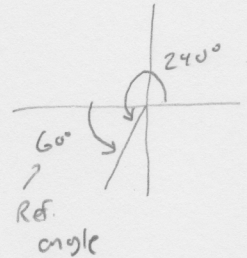


$$(1.) \quad \vec{v} = (\cos(30^\circ), \sin(30^\circ)) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$$

$$(2.) \quad \vec{v} = 3 (\cos(240^\circ), \sin(240^\circ)) = 3(-\cos(60^\circ), -\sin(60^\circ)) \\ = (-3(\frac{1}{2}), -3(\frac{\sqrt{3}}{2}))$$

$\uparrow$ Length of 3
 $\nwarrow$ unit vector forming 240° angle



$$(3.) (a) \quad \vec{u} = \frac{1}{\|\vec{v}\|} \vec{v} = \frac{1}{\|(1, 3, 4)\|} (1, 3, 4)$$

$$= \frac{1}{\sqrt{1+9+16}} (1, 3, 4) = \left(\frac{1}{\sqrt{26}}, \frac{3}{\sqrt{26}}, \frac{4}{\sqrt{26}}\right)$$

$$(b) \quad \|2\vec{v} - \vec{w}\| = \|2(1, 3, 4) - (1, -1, 0)\| = \|(2, 6, 8) - (1, -1, 0)\| \\ = \|(1, 7, 8)\| = \sqrt{1+49+64} = \sqrt{114}$$

$$(c) \quad \cos \theta = \frac{\vec{v} \cdot \vec{w}}{\|\vec{v}\| \|\vec{w}\|} = \frac{(1, 3, 4) \cdot (1, -1, 0)}{\|(1, 3, 4)\| \|(1, -1, 0)\|} = \frac{1 + -3 + 0}{\sqrt{1+9+16} \sqrt{1+1+0}} \\ = \frac{-2}{\sqrt{52}} \quad \text{So} \quad \theta = \cos^{-1}\left(\frac{-2}{\sqrt{52}}\right)$$

$$(d) \quad \vec{n} = (1, 3, 4) \quad P(1, 1, 1)$$

$$a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$$

$$1(x-1) + 3(y-1) + 4(z-1) = 0$$

(4.) $P(0, 1, 2)$ and $Q(1, 2, 3)$ Line

$$(a) \begin{cases} x = 0 + t \\ y = 1 + t \\ z = 2 + 2t \end{cases}$$

$$\begin{matrix} \uparrow & \uparrow \\ P & Q \end{matrix} \quad \vec{v} = \vec{PQ}$$

$$\vec{v} = \vec{PQ} = (1-0, 2-1, 3-2) = (1, 1, 1)$$

$$(b) \begin{cases} x = -5 + t \\ y = 1 + 2t \\ z = 3 + 3t \end{cases}$$

(5.) (a) $P(2, 2, 2)$, $Q(1, 2, 3)$ AND $R(1, -1, 0)$

$$\vec{v} = \vec{PQ} = (1-2, 2-2, 3-2) = (-1, 0, 1)$$

$$\vec{w} = \vec{PR} = (1-2, -1-2, 0-2) = (-1, -3, -2)$$

PARAMETRIC

$$L \begin{cases} x = 2 + -t + -5s \\ y = 2 + 0t + -3s \\ z = 2 + 1t + -2s \end{cases}$$

POINT
-NORMAL

$$\vec{n} = \vec{v} \times \vec{w} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 0 & 1 \\ -1 & -3 & -2 \end{vmatrix}$$

$$= \hat{i} \begin{vmatrix} 0 & 1 \\ -3 & -2 \end{vmatrix} - \hat{j} \begin{vmatrix} -1 & 1 \\ -1 & -2 \end{vmatrix} + \hat{k} \begin{vmatrix} -1 & 0 \\ -1 & -3 \end{vmatrix}$$

$$= \hat{i}(0+3) - \hat{j}(2+1) + \hat{k}(3-0)$$

$$= 3\hat{i} - 3\hat{j} + 3\hat{k} = (3, -3, 3)$$

$$a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$$

$$3(x-2) - 3(y-2) + 3(z-2) = 0$$

⑤(b) ORIGIN & perp. to $\vec{V} = (1, 3, 3)$

POINT is
(0, 0, 0)

SO $\vec{n} = (1, 2, 3)$

PARAMETRIC

$$P \begin{cases} x = 0 + 3t + 2s \\ y = 0 + 0 - s \\ z = 0 + -t \end{cases}$$

two vectors perp to $\vec{n}$ are

$$\vec{u} = (3, 0, -1)$$

$$\vec{w} = (2, -1, 0)$$

STANDARD

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

$$1(x - 0) + 2(y - 0) + 3(z - 0) = 0$$

(c) POINT (1, 2, 3)

vectors

$$\vec{V} = (1, 0, -3) \text{ AND } \vec{W} = (2, 1, 3)$$

$$\vec{n} = (1, 0, -3) \times (2, 1, 3)$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & -3 \\ 2 & 1 & 3 \end{vmatrix} = (+3, -9, 1-0)$$

$$= (3, -9, 1)$$

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

$$3(x - 1) + (-9)(y - 2) + 1(z - 3) = 0$$

⑥ (a) $P_1: x - 2y + z = 12$
 $P_2: 3x - 3y + z = 4$

$$\vec{n}_1 = (1, -2, 1)$$

$$\vec{n}_2 = (3, -3, 1)$$

(b) $\cos \theta = \frac{\vec{n}_1 \cdot \vec{n}_2}{\|\vec{n}_1\| \|\vec{n}_2\|} = \frac{(1, -2, 1) \cdot (3, -3, 1)}{\|(1, -2, 1)\| \|(3, -3, 1)\|} = \frac{3 + 6 + 1}{\sqrt{1+4+1} \sqrt{9+9+1}} = \frac{10}{\sqrt{6}\sqrt{19}}$

$$\theta = \cos^{-1} \left(\frac{10}{\sqrt{6}\sqrt{19}} \right)$$

(c) $(4, 2, 12) \in P_1$, $(0, 0, 4) \in P_2$

(d) $\left[\begin{array}{ccc|c} 1 & -2 & 1 & 12 \\ 3 & -3 & 1 & 4 \end{array} \right] \rightarrow R_2 \dots \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -1/3 & -28/3 \\ 0 & 1 & -2/3 & -32/3 \end{array} \right] \xrightarrow{z=t} \begin{cases} x - \frac{1}{3}z = -\frac{28}{3} \\ x - \frac{1}{3}t = -\frac{28}{3} \\ x = -\frac{28}{3} + \frac{1}{3}t \end{cases}$

$$S = \left\{ \begin{bmatrix} -\frac{28}{3} + \frac{1}{3}t \\ -\frac{32}{3} + \frac{2}{3}t \\ t \end{bmatrix} : t \in \mathbb{R} \right\}$$

(e) $x = -\frac{28}{3} + \frac{1}{3}t$

$$y = -\frac{32}{3} + \frac{2}{3}t$$

$$z = t$$

⑦ The planes are parallel since their NORMAL vectors ARE parallel. $\vec{n}_1 = (1, -2, 1)$, $\vec{n}_2 = (3, -6, 3)$

Note $3\vec{n}_1 = \vec{n}_2$ so $\vec{n}_1 \parallel \vec{n}_2$.

⑧ $P(1, 0, 2)$ $Q(0, 0, 2)$ $S(1, 1, 1)$ | $\vec{v} = \vec{PQ} = (-1, 0, 0)$ $\vec{w} = \vec{SQ} = (-1, -1, 1)$

$$\vec{n} = \vec{v} \times \vec{w} = (-1, 0, 0) \times (-1, -1, 1) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 0 & 0 \\ -1 & -1 & 1 \end{vmatrix} = (0, 1, 1)$$

PARAMETRIC

POINT NORMAL

$$a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$$

$$0(x-1) + 1(y-0) + 1(z-2) = 0$$

$\vec{P}$ AND $\vec{n}$

$$x = 1 + -1t \quad 1S$$

$$y = 0 + 0t \quad -1S$$

$$z = 2 + 0t + 1S$$

$\vec{P}$ $\vec{v}$ $\vec{w}$

9. $\vec{n} = (1, -4, 2, 1)$

10. For you.

11. $\vec{v} = (1, 2, 3)$ $\vec{w} = (0, 1, 1)$
 (a) $\text{proj}_{\vec{w}} \vec{v} = \frac{\vec{v} \cdot \vec{w}}{\vec{w} \cdot \vec{w}} \vec{w} = \frac{(1, 2, 3) \cdot (0, 1, 1)}{(0, 1, 1) \cdot (0, 1, 1)} (0, 1, 1) = \frac{5}{2} (0, 1, 1)$
 $= (0, \frac{5}{2}, \frac{5}{2})$

(b) parallel comp = $\text{proj}_{\vec{w}} \vec{v} = (0, \frac{5}{2}, \frac{5}{2})$

perp. comp = $\vec{v} - \text{proj}_{\vec{w}} \vec{v} = (1, 2, 3) - (0, \frac{5}{2}, \frac{5}{2})$
 $= (1, -\frac{1}{2}, +\frac{1}{2})$

12. Just read and convince yourself $\mathbb{R}^3$ satisfies prop. 1-10.

13. (a) $(0, 4) \oplus (-2, 3) = (0 + -2 + 1, 4 + 3 + 1) = (-1, 8)$

$2 \odot (1, 1) = (2, 2)$

(b) $(0, 0) \oplus (x, y) = (x + 1, y + 1) \neq (x, y)$

so $\vec{0} \neq (0, 0)$.

(c) $\vec{0} = (-1, -1)$. That is $(-1, -1) \oplus (x, y) = (x, y)$

Proof. Let $\vec{v} = (x, y) \in \mathbb{R}^2$. Note

$(-1, -1) \oplus (x, y) = (x + (-1) + 1, y + (-1) + 1) = (x, y)$

Similarly $(x, y) \oplus (-1, -1) = (x, y)$. $\square$

(d) We are given (x, y) AND WANT TO FIND (a, b) s.t.

$(x, y) \oplus (a, b) = \vec{0} = (-1, -1)$

(13d) (CONTINUED)

So $(x+a+1, y+b+1) = (-1, -1)$

$$\left. \begin{array}{l} x+a+1 = -1 \\ y+b+1 = -1 \end{array} \right\} \Rightarrow \begin{array}{l} a = -x-2 \\ b = -y-2 \end{array}$$

Claim. If $\vec{v} = (x, y)$ then $-\vec{v} = (-x-2, -y-2)$.

Proof. Let $\vec{v} = (x, y)$, $x, y \in \mathbb{R}$. Define $\vec{w} = (-x-2, -y-2)$.

We will show $\vec{w} = -\vec{v}$. That is we will

show $\vec{v} \oplus \vec{w} = \vec{w} \oplus \vec{v} = \vec{0} = (-1, -1)$

Note $\vec{v} + \vec{w} = (x, y) \oplus (-x-2, -y-2) = (x + (-x-2)+1, y + (-y-2)+1)$

$$= (-1, -1) = \vec{0}, \quad \square$$

(e) $(a+b) \odot \vec{v} = a \vec{v} \oplus b \vec{v}$
use $a = 1$, $b = -1$, $\vec{v} = (1, 2)$

$$\begin{aligned} (a+b) \odot \vec{v} &= (1+(-1)) \odot (1, 2) \\ &= 0 \odot (1, 2) = (0, 0) \end{aligned}$$

$$\begin{aligned} a \odot \vec{v} + b \odot \vec{v} &= 1 \odot (1, 2) \oplus (-1) \odot (1, 2) \\ &= (1, 2) \oplus (-1, -2) = (1, 1) \end{aligned}$$

Note $(0, 0) \neq (1, 1)$. So

$$(a+b) \odot \vec{v} \neq a \odot \vec{v} \oplus b \odot \vec{v}$$

You try & find another
Look at the distributive laws.

(14.) Look it up in your Notes

(15.) Note W is not a subspace

(a) note $\vec{0} \in W$. Since $\vec{0} = (0, 0, 0)$ AND

$$a + b + c = 1 \\ 0 + 0 + 0 = 1 \leftarrow \text{fails to be true.}$$

(b) $x + y + z = 1$ is a plane in standard form.

(c) $\vec{n} = (1, 1, 1)$

$\vec{v} = (1, -1, 0)$

$\vec{w} = (0, 1, -1)$

$P(2, 3, -4) \in P$

$$x = 2 + t$$

$$y = 3 + t + s$$

$$z = -4 + \quad -s$$

$\uparrow$

P

$\uparrow$

$\vec{v}$

$\uparrow$

$\vec{w}$

(16.) (2) (i) Show if $\vec{v}, \vec{w} \in W$ then $\vec{v} + \vec{w} \in W$.

Proof. Let $\vec{v}, \vec{w} \in W$. Then $\vec{v} = (a, b, c)$ and $\vec{w} = (d, e, f)$ where

$$a - 3b - c = 0 \quad \text{AND} \quad d - 3e - f = 0, (*)$$

Note $\vec{v} + \vec{w} = (a, b, c) + (d, e, f) = (a+d, b+e, c+f)$

Note $x - 3y - z = (a+d) - (3)(b+e) - (c+f)$

$$= (a - 3b - c) +$$

$$= 0 + 0 \quad \text{by } (*)$$

$$= 0$$

So $\vec{v} + \vec{w} \in W, \square$

(2) Show if $k \in \mathbb{R}, \vec{v} \in W$ then $k\vec{v} \in W$.

$\square$ (for you.)

(17) $S = \{(1, 2, 1), (0, 1, 2), (0, -1, 0)\}$

(a) $\begin{bmatrix} 1 & 0 & 0 & | & 0 \\ 2 & 1 & -1 & | & 0 \\ 1 & 2 & 0 & | & 0 \end{bmatrix} \rightarrow R.R. \rightarrow \begin{bmatrix} 1 & 0 & 0 & | & 0 \\ 0 & 1 & -1 & | & 0 \\ 0 & 0 & 1 & | & 0 \end{bmatrix}$

So $S = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$. The ONLY solution is the trivial sol'n, so S is INDEPENDENT.

(b) $(2, 2, 2) \in \text{SPAN}(S)$

$\begin{bmatrix} 1 & 0 & 0 & | & 2 \\ 2 & 1 & -1 & | & 2 \\ 1 & 2 & 0 & | & 2 \end{bmatrix} \rightarrow R.R. \rightarrow \begin{bmatrix} 1 & 0 & 0 & | & 2 \\ 0 & 1 & -1 & | & 0 \\ 0 & 0 & 1 & | & 2 \end{bmatrix}$

So $2 \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} + 0 \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} + 2 \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$.

(c) Does S span $\mathbb{R}^3$? Yes. Since

$\begin{bmatrix} 1 \\ \frac{1}{\sqrt{2}} \\ 1 \end{bmatrix}, \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ \frac{1}{\sqrt{2}} \\ 1 \end{bmatrix} \rightarrow R.R. \text{ Has } 3 \text{ leading ones}$

$\rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(18) Just like #17 except we will look at coefficients to form our vectors. Can you see why? So

$x \sim \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad x+2 \sim \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad x^3-x-1 \sim \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix} \quad \text{what is } x^3?$

Repeat work for #17 with this adjustment.

(18d) we will need 4 leading ones to span.

(19.) $B = \{ (1, 2, 1), (0, 1, 2), (0, -1, 0) \}$

(a) To TEST IF B IS A BASIS IT MUST BE
(1) LIN. IND AND (2) SPANNING.

(1) TO TEST FOR LIN IND

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 2 & 1 & -1 & 0 \\ 1 & 2 & 0 & 0 \end{array} \right] \rightarrow \dots \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right]$$

↑ This tells us yes B is lin. ind.

(2) SPAN.

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 \\ 2 & 1 & -1 \\ 1 & 2 & 0 \end{array} \right] \rightarrow \text{P.R.} \left[\begin{array}{ccc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right]$$

3 leading ones, says
Yes B spans $\mathbb{R}^3$

So yes + yes means B is a basis for $\mathbb{R}^3$.

(b) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 2 & 1 & -1 & 0 \\ 1 & 2 & 0 & -1 \end{array} \right] \xrightarrow{\text{RR}} \dots \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array} \right]$ Yes $(1, 0, -1) \in \text{SPAN}(B)$

AND $(1, 0, -1) = (1) \underset{\uparrow}{\vec{v}_1} + (-1) \underset{\uparrow}{\vec{v}_2} + (1) \underset{\uparrow}{\vec{v}_3} = (1, -1, 1)_B$

$$\begin{matrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} & \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} & \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \end{matrix}$$

(c) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & a \\ 2 & 1 & -1 & b \\ 1 & 2 & 0 & c \end{array} \right] \xrightarrow{\text{RR}} \dots \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & a \\ 0 & 1 & 0 & -\frac{1}{2}a - \frac{1}{2}c \\ 0 & 0 & 1 & \frac{3}{2}a - b + \frac{1}{2}c \end{array} \right]$

so $(a, b, c) = (a, -\frac{1}{2}a - \frac{1}{2}c, \frac{3}{2}a - b + \frac{1}{2}c)_B$

(20)

$$2x_1 - 2x_2 + 4x_3$$

$$-6x_5 = 0$$

$$x_3 + 6x_4$$

$$= 0$$

$$\left[\begin{array}{ccccc|c} 2 & -2 & 4 & 0 & -6 & 0 \\ 0 & 0 & 1 & 6 & 0 & 0 \end{array} \right] \rightarrow \text{R.R.} \rightarrow \left[\begin{array}{ccccc|c} 1 & -1 & 0 & -12 & -3 & 0 \\ 0 & 0 & 1 & 6 & 0 & 0 \end{array} \right]$$

$x_2 = s \quad x_4 = t \quad x_5 = u$

$$(a) \quad S = \left\{ \begin{bmatrix} s + 12t + 3u \\ -s \\ -6t \\ t \\ u \end{bmatrix} : s, t, u \in \mathbb{R} \right\}$$

$$(b) \quad = \left\{ \begin{bmatrix} s \\ s \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 12t \\ 0 \\ -6t \\ t \\ 0 \end{bmatrix} + \begin{bmatrix} 3u \\ 0 \\ 0 \\ 0 \\ u \end{bmatrix} : s, t, u \in \mathbb{R} \right\} = \left\{ s \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} 12 \\ 0 \\ -6 \\ 1 \\ 0 \end{bmatrix} + u \begin{bmatrix} 3 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} : s, t, u \in \mathbb{R} \right\}$$

$$\beta = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 12 \\ 0 \\ -6 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\} \text{ is the basis.}$$

$$(c) \quad \text{So } \dim(S) = 3.$$

(21.) Might be extra credit?

(23) $W: x - 2y + z = 0$

(a) $P(1, 0, -1), Q(2, 1, 0), R(0, 1, 2) \in W$

So $\vec{v} = \vec{PQ} = (1, 1, 1), \vec{w} = \vec{PR} = (-1, 1, 3)$ are in the plane

$$\left\{ \begin{array}{l} x = 1 + 1t + (-1)s \\ y = 0 + 1t + (1)s \\ z = -1 + 1t + 3s \end{array} \right.$$

$\uparrow$
 $P \quad \vec{PQ} \quad \vec{PR}$

(b) $B = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix} \right\}$

(c) $x - 2y + z = 0$

$$[1 \ -2 \ 1 \ | \ 0] \rightarrow \text{R.R.} \rightarrow [1 \ -2 \ 1 \ | \ 0] \rightarrow \begin{array}{l} x - 2y + z = 0 \\ y = t \quad z = s \quad x - 2t + s = 0 \\ x = 2t - s \end{array}$$

$$S = \left\{ \begin{bmatrix} 2t - s \\ t \\ s \end{bmatrix} : t, s \in \mathbb{R} \right\}$$

(24) [For EXTRA CREDIT ON TEST.] SAME TECHNIQUE AS IN $\mathbb{R}^3$.

(25) $\begin{bmatrix} -1 & 2 & 0 & 3 & 0 \\ 2 & 1 & 1 & -1 & 1 \\ 1 & 3 & 1 & 2 & 1 \end{bmatrix} \rightarrow \text{R.R.} \begin{bmatrix} 1 & 0 & 2/5 & -1 & 2/5 \\ 0 & 1 & 1/5 & 1 & 1/5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

(a) $\text{col}(A) = \text{SPAN} \left\{ \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} \right\}$

(b) $\dim(\text{col}(A)) = 2$

(25 c)

(c) $\text{Null}(A)$

$$\left[\begin{array}{cccc|c} -1 & 2 & 0 & 3 & 0 \\ 2 & 1 & 1 & -1 & 0 \\ 1 & 3 & 1 & 2 & 0 \end{array} \right] \rightarrow \text{R.M.} \rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 2/5 & -1 & 2/5 \\ 0 & 1 & 1/5 & 1 & 1/5 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$x_3 = s \quad \uparrow \quad x_4 = t \quad \uparrow \quad x_5 = u$

$$S = \left\{ \begin{bmatrix} -2/5 s + 1 t - 2/5 u \\ -1/5 s - 1 t - 1/5 u \\ s \\ t \\ u \end{bmatrix} : s, t, u \in \mathbb{R} \right\}$$

$$= \left\{ s \begin{bmatrix} -2/5 \\ -1/5 \\ 1 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + u \begin{bmatrix} -2/5 \\ -1/5 \\ 0 \\ 0 \\ 1 \end{bmatrix} : s, t, u \in \mathbb{R} \right\}$$

$$B = \left\{ \begin{bmatrix} -2/5 \\ -1/5 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2/5 \\ -1/5 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\} \quad \boxed{\text{Null}(A) = S = \text{Span}(B)}$$

(d) $\dim(\text{Null}(A)) = 3$

(26.) (a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix} \rightarrow \text{R.R.} \rightarrow \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

$B = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \right\}$

$\text{RANGE}(T) = \text{Col}(A) = \text{SPAN}(B)$

(c) Same matrix WITH 0's

$\left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$

$x - z = 0$

$x - t = 0$

$x = t$

$z = t$

$y + z = 0$
 $y = -t$

$B_2 = \left\{ \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} \right\}$

SO $\text{NULL}(T) = \text{SPAN} \left\{ \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} \right\}$

$S = \left\{ \begin{bmatrix} t \\ -t \\ t \end{bmatrix} : t \in \mathbb{R} \right\}$
 $= \left\{ t \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} : t \in \mathbb{R} \right\}$

(d) $\dim(\text{NULL}(T)) = \underline{1}$

(e) $\text{RANK}(T) = 2$, $\text{NULLITY}(T) = 1$, $\dim \text{ of Domain} = 3$

$2 + 1 = 3$