Math 3160 - Test 2 Version 2

Name:____

No calculators, no phones, no electronics allowed. Remember, I am grading your work. I must see how you came to the conclusion.

1. Let $P(1, 2, 3, 1)$, $Q(1, 0, 1, 4)$, $R(0, 0, 1, 0)$ and $S(0, 1, 1, 1)$ be points in $\mathbb{R}^4$.

   (a) Find the angle between the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$.

   (b) Find the normal vector of the hyperplane containing P, Q, R and S.

2. Define the planes P_1 and P_2 as follows:

$$P_1 : x - y + z = 8$$

$$P_2 : x + y + z = 4$$

(a) Compute the angle between P_1 and P_2.

(b) What is the equation of the line of intersection of P_1 and P_2?

3. Let $\mathbf{v} = (1, 2, 0)$ and $\mathbf{w} = (0, -2, 1)$

   (a) Compute the projection of $\mathbf{v}$ onto $\mathbf{w}$.

   (b) Compute the parallel component of $\mathbf{v}$ onto $\mathbf{w}$.

   (c) Compute the cooresponding perpendicular component of $\mathbf{v}$ onto $\mathbf{w}$.

4. Two step subspace test. Let

$$W = \{(a-b, b, 2b, a) \in \mathbb{R}^4 : a, b \in \mathbb{R}\}.$$

Use the two step subspace test to show $(W, +, \cdot)$ is a subspace of $\mathbb{R}^4$.

5. Let $B = \{(1,0,1), (-1,1,0), (2,2,0)\}$. Is the set of vectors independent? Show your work.

6. Let $\mathbf{v_1} = (1, 2, -3, 1), \mathbf{v_2} = (1, -3, 2, 0), \mathbf{v_3} = (-1, 2, 0, 0)$, and $\mathbf{v_4} = (2, 6, -5, 3)$, $B = \{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}, \mathbf{v_4}\}$. Show B is not independent, by finding a nontrivial linear combination of the vectors equal to the zero vector.

7. Let $B = \{(1,2,3),(1,3,4),(1,4,4)\}$.

   (a) Does B span $\mathbb{R}^3$? Show your work.

   (b) Show $(2,3,7) \in \text{span}(B)$ by finding a linear combination of the vectors in B equal to $(2,3,7)$.

8. The linear transformation T is given by the formula

$$T\left(\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}\right) = \begin{bmatrix} x + y + 4z + 3w \\ x + y + 2z + w \\ -x - y + 2z + 3w \end{bmatrix}.$$

(a) What are the domain and codomain of T?

(b) Find the matrix, A, to represent the linear transformation T.

(c) Compute the basis for the column space of A, COL(A).

(d) Compute the dimension of the COL(A). The dimension of the column space is called the rank.

9. The linear transformation T is given by the formula

$$T(\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}) = \begin{bmatrix} x+y+4z+3w \\ x+y+2z+w \\ -x-y+2z+3w \end{bmatrix}.$$

(a) Compute the basis for the null space of A, NULL(A).

(b) Compute the dimension of the NULL(A). The dimension of the null space is called the nullity.

(c) What is the dimension of the domain of T and the codomain of T? What do you expect the dimension of the Range of T to be?

ec. Let V, W be vector spaces. Prove the following.
If $T : V \to W$ is a linear transformation then the Range(T) is a subspace of W.