Math 3160 - Test 1 Review

1 Systems of Linear Equations

1. Solve the following systems of linear equations using row reduction.

(a) $$\begin{cases} x_1 - 2x_2 - 6x_5 = 0 \\ x_2 + x_3 + 6x_4 = 5 \\ 2x_2 + 6x_4 + x_5 = 4 \\ x_2 - x_3 + x_5 = -1 \end{cases}$$

(b) $$\begin{cases} 2x_1 - 2x_2 + 4x_3 = 2 \\ x_3 = 0 \\ x_1 + x_2 + 2x_3 = 0 \end{cases}$$

(c) $$\begin{cases} 2x_1 - 2x_2 + 4x_3 = 2 \\ -x_1 - x_2 + 3x_3 = 2 \\ x_1 - 3x_2 + 7x_3 = 2 \end{cases}$$

2. Solve the following systems of linear equations by setting up problem as a matrix problem and by finding an inverse matrix.

(a) $$\begin{cases} 2x_1 - 2x_2 + 4x_3 = 2 \\ -x_2 + 3x_3 = 2 \\ -3x_2 + 7x_3 = 2 \end{cases}$$

(b) $$\begin{cases} 2x_1 - 2y = 2 \\ -x_1 - 3y = 2 \end{cases}$$

2 Matrices, Determinants and Cramer's Rule

3. Let $A = \begin{bmatrix} 2 & -2 & 0 \\ -1 & 4 & 1 \\ 0 & 4 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -2 & 0 & 0 \\ 2 & -2 & 1 & 2 \\ 2 & -2 & 0 & 3 \\ 2 & -2 & 5 & 4 \end{bmatrix}$, and $C = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

   (a) Find the determinant of the matices A, B, C and D
   (b) Compute D^3 and D^{-1}.
   (c) Compute C^TC. What kind of matrix is C^TC?

4. Solve the following equations for X assuming any matrix has an inverse. Let A, B, C, X be $n \times n$ matrices and let $\mathbf{u}$ be an $n \times 1$ vector.

(a) $AX = BX - A$

(b) $AX = 2X - A$

(c) $A\mathbf{u} = 2\mathbf{u} + B$

5. Solve the following using Cramer's rule.

(a) $$\begin{cases} 2x_1 & -2x_2 & +4x_3 & = 2 \\ & -x_2 & +3x_3 & = 2 \\ & -3x_2 & +7x_3 & = 2 \end{cases}$$

(b) $$\begin{cases} 2x_1 & -2y & = 2 \\ -x_1 & -3y & = 2 \end{cases}$$

6. Solve the following systems of linear equation using $A\mathbf{x} = \mathbf{b}$ and the inverse of A.

(a) $$\begin{cases} x_1 & -2x_2 & = 0 \\ x_1 & x_2 & = 5 \end{cases}$$

(b) $$\begin{cases} x_1 & -2x_2 & & = 0 \\ & x_2 & +x_3 & = 5 \\ & 2x_2 & & = 4 \end{cases}$$

3 Basic Transformations

7. Write the matrix for the following transformations described below.

(a) $T : \mathbb{R}^2 \to \mathbb{R}^2$ where the plane is rotated by $45°$ counter-clockwise.

(b) $T : \mathbb{R}^2 \to \mathbb{R}^2$ where the plane is reflected about the x-axis.

(c) $T : \mathbb{R}^2 \to \mathbb{R}^2$ where the x-axis is contracted by half and the y-axis is dilated by 2.

(d) $T : \mathbb{R}^2 \to \mathbb{R}^2$ where the plane is rotated by $30°$ counter-clockwise and then reflected about the x-axis.

(e) $T : \mathbb{R}^2 \to \mathbb{R}^2$ where the plane is reflected about the x-axis and then rotated by $30°$ counter-clockwise.

(f) $T : \mathbb{R}^3 \to \mathbb{R}^3$ where the x-axis is contracted by half and the z-axis is dilated by 2.

4 Vectors

8. Let $\mathbf{v} = (1, 3, 4)$ and $\mathbf{w} = (1, -1, 0)$ be vectors in $\mathbb{R}^3$. and let $P(1, 1, 1)$ and $Q(0, -4, 0)$ be two points in $\mathbb{R}^3$.

   (a) Find the vector $\overrightarrow{PQ}$ and a unit vector in the same direction as $\overrightarrow{PQ}$.

   (b) Find a vector that is parallel to $\mathbf{v}$ and unit.

   (c) Compute $\|2\mathbf{v} - \mathbf{w}\|$.