

Math 3160 - Gram-Schmidt, Orthogonality, Least Squares, and QR Decomposition

1. Find the Gram-Schmidt orthogonalization of the following vectors using the usual inner product.

$$\mathbf{v}_1 = (0, 0, 3), \mathbf{v}_2 = (1, 0, -2), \mathbf{v}_3 = (1, 2, 3)$$

2. Find the Gram-Schmidt orthogonalization of the following vectors

$$f_1(x) = 1, f_2(x) = 1 - x, f_3(x) = 1 + x^2$$

with the following inner product.

$$\langle f(x), g(x) \rangle = \int_0^1 f(x)g(x) dx$$

3. Find the Gram-Schmidt orthogonalization of the column vectors of A using the usual inner product. And find the QR decomposition of A

$$A = \begin{bmatrix} 1 & 4 & 0 \\ 0 & 3 & 0 \\ 0 & 3 & 5 \end{bmatrix}$$

4. For this problem we will attempt to find the Least Squares approximation to a line $y = a_0 + a_1x$ for the points $(0, 2)$, $(1, 3)$, $(2, 7)$ and $(3, 6)$.

- (a) Graph the points and guess what the function may be?
- (b) My guess is $y = x + 3$ (not a very good guess). Compute the error for my function and yours, which is better? Use

$$E(f) = \sum (y_i - f(x_i))^2$$

as we did in class.

- (c) What are the matrices A , $\mathbf{u}$ and $\mathbf{y}$ to get

$$A\mathbf{u} = \mathbf{y}.$$

- (d) Compute

$$\mathbf{u} = (A^T A)^{-1} A^T \mathbf{y}.$$

- (e) What is the least squares function $f(x) = a_0 + a_1x$.

5. For this problem we will attempt to find the Least Squares approximation to a quadratic $y = a_0 + a_1x + a_2x^2$ for the points $(0, 2)$, $(1, 3)$, $(2, 7)$ and $(3, 6)$.

- (a) Graph the points and guess what the function may be?
(b) My guess is $y = x^2 + 2$ (not a very good guess). Compute the error for my function and yours, which is better? Use

$$E(f) = \sum (y_i - f(x_i))^2$$

as we did in class (hint my error is 22).

- (c) What are the matrices A , $\mathbf{u}$ and $\mathbf{y}$ to get

$$A\mathbf{u} = \mathbf{y}.$$

- (d) Compute

$$\mathbf{u} = (A^T A)^{-1} A^T \mathbf{y}.$$

- (e) What is the least squares function $f(x) = a_0 + a_1x + a_2x^2$.