Math 6250: Test 1

Name:____

1. Find the limit and prove your answer.

$$\lim_{n\to\infty} \frac{3n+2}{5n+4}$$

2. Prove $2^n > n$ for all $n \in \mathbb{N}$.

3. Do one of the following.

   - For the following relation defined on $\mathbb{Z}$

     $$a \sim b \text{ if and only if } 3|a+2b$$

     prove $\sim$ is an equivalence relation.
   - For the following relation defined on $\mathbb{Z}$

     $$a \sim b \text{ if and only if } 3|a+2b$$

     prove addition is well defined.

4. Do one of the following.

   - Prove $f : (0, 2) \to (-3, 5)$ where $f(x) = 2x^2 - 3$ is a bijection.
   - Select one and prove $\mathbb{Q} \sim \mathbb{N}$, $\mathbb{Z} \sim \mathbb{N}$, or $\mathbb{R} \not\sim \mathbb{N}$.

5. Prove: Let $\alpha = \sup(A)$. If $\varepsilon > 0$ then there is some $x \in A$ so that $\alpha - \varepsilon < x \leq \alpha$.

6. Solve $z^4 = 1$ where $z \in \mathbb{C}$.

7. Prove if a sequence is convergent then it is bounded.

ec 2. Let $A \subseteq \mathbb{R}$, $A \neq$ and A is bounded above. So the $sup(A)$ exists. Say $\alpha = sup(A)$. Show if $\alpha \notin A$, there exists an increasing sequence, (a_n) in A so that

$$\lim a_n = \alpha$$

$$a_n = \frac{2a_n + 1}{a_n + 1}, a_1 = 1.$$

So

$$a_2 = \frac{2a_1 + 1}{a_1 + 1} = \frac{2(1) + 1}{(1) + 1} = \frac{3}{2}.$$

(a) Compute a_3 and a_4.

(b) Show that $a_n < a_{n+1}$ for all $n \in \mathbb{N}$. That is, show (a_n) is increasing.

(c) Since (a_n) is increasing we have $a_1 = 1 < a_2 < \cdots$. That is $1 < a_n$ for all $n \in \mathbb{N}$. Show $a_n < 10$ for all $n \in \mathbb{N}$.

For both problems I used induction and the fact that

$$a_n = \frac{2a_n + 1}{a_n + 1} = 2 - \frac{1}{a_n + 1}.$$

ec 1. Define the sequence as follows

$$a_n = \frac{2a_n + 1}{a_n + 1}, a_1 = 1.$$

So

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