Math 3330 **Test 1 Review** Fall 2024

1. To answer the following questions use the function given parametrically
$$x = 2t^2, y = 5t + 1$$
   (a) Graph
   (b) Find the tangent line to the function at $t = 1$.
   (c) Compute the area under the curve from $t = 0$ to $t = 2$.
   (d) Translate into rectangular coordinates.

2. To answer the following questions use the function given parametrically
$$x = 2\cos(t), y = \sin(t)$$
   (a) Graph
   (b) Find the tangent line to the function at $t = \pi/6$.
   (c) Set up the integral to compute the area under the curve from $t = 0$ to $t = \pi/2$. You do not need to compute the integral.
   (d) Translate into rectangular coordinates.

3. To answer the following questions use the function given parametrically
$$x = \cos(e^{2t}), y = \sin(e^{2t})$$
   (a) Graph.
   (b) Compute the arclength from $t = 0$ to $t = 4$.
   (c) Translate into rectangular coordinates.

4. To answer the following questions use the function given in polar coordinates
$$r = 2 + \cos(\theta)$$
   (a) Graph.
   (b) Find the tangent line to the function at $\theta = \pi/4$.
   (c) Translate into rectangular coordinates.

5. To answer the following questions use the function given in polar coordinates
$$r = 2\cos(2\theta)$$
   (a) Graph
   (b) Find the tangent line to the function at $\theta = \pi/6$.
   (c) Translate into rectangular coordinates.

8. Graph $r = 2\cos(2\theta)$.

9. Find all two-dimensional vectors a orthogonal to vector $\mathbf{b} = \langle 5, -6 \rangle$. Express the answer by using standard unit vectors.

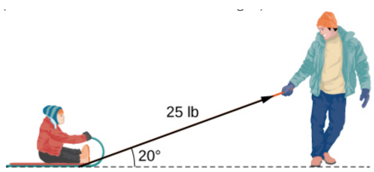
10. Consider points $A(3, -1, 2)$, $B(2, 1, 5)$, and $C(1, -2, -2)$. Find the area of parallelogram ABCD and find the area of triangle ABC.

11. Determine the real number α such that $\mathbf{u} \times \mathbf{v}$ and i are orthogonal, where $\mathbf{u} = 3i + j - 5k$ and $\mathbf{v} = 4i - 2j + \alpha k$.

12. A sled is towed using a force of 1600 N. The rope used to pull the sled makes an angle of 20° with the horizontal.

    (a) Find the parallel and perpandicular components of the force vector.

    (b) Find the work done in towing the sled 200 m. Express the answer in joules ($1J = 1Nm$).



13. Find the equation of the line that

    (a) goes through the points $A(1, 2, 0)$ and $B(0, 1, 1)$.

    (b) goes through the point $A(1, 2, 0)$ and is parallel to the vector $\mathbf{v} = \langle 5, 2, 3 \rangle$.

    (c) goes through the point $A(1, 2, 0)$ and is perpendicular to the vector $\mathbf{v} = \langle 5, 2, 3 \rangle$ (there are many answers to this question).

    (d) Is the point $(1, 1, 1)$ on the line from Problem 13a?

14. Do the lines defined below intersect? If yes, where? If they intersect, at what angle?

$$L_1 : \begin{cases} x = & 2 & +3t \\ y = & 1 & +t \\ z = & 7 & \end{cases} \qquad \text{and} \qquad L_2 : \begin{cases} x = & & t \\ y = & 4 & -t \\ z = & 2 & +t \end{cases}$$

15. Do the lines defined below intersect? If yes, where? If they intersect, at what angle?

$$L_1 : \begin{cases} x = & 3 & -3t \\ y = & 32 & +2t \\ z = & & +3t \end{cases} \qquad \text{and} \qquad L_2 : \begin{cases} x = & -3 & +t \\ y = & 7 & +3t \\ z = & 6 & \end{cases}$$

16. Find the equation (vector equation and standard equation) of the plane that

    (a) goes through the three points $A(1,2,0)$, $B(0,1,1)$ and $C(0,0,0)$

    (b) Is parallel to the two vectors $\mathbf{v} = \langle 5,2,3\rangle$ and $\mathbf{w} = \langle 1,2,0\rangle$. and goes through the point $A(1,2,0)$.

    (c) Is perpandicular to the vector $\mathbf{v} = \langle 5,2,3\rangle$ and and goes through the point $A(1,2,0)$.

17. Do the planes defined below intersect? If yes, where? If they intersect, at what angle?

$$P_1 : \begin{cases} x = & 1 & -3t & \\ x = & 2 & +2t & -s \\ z = & & +3t & +s \end{cases} \qquad \text{and} \qquad P_2 : \begin{cases} x = & 0 & +t & +s \\ x = & 0 & +3t & -s \\ z = & 1 & & +2s \end{cases}$$

18. Do the planes defined below intersect? If yes, where? If they intersect, at what angle?

$$P_1 : x - 2y + z = 2 \qquad \text{and} \qquad P_2 : \begin{cases} x = & 0 & +t & +s \\ x = & 0 & +3t & -s \\ z = & 1 & & +2s \end{cases}$$

19. Do the plane and line defined below intersect? If yes, where? If they intersect, at what angle?

$$P_1 : x - 2y + z = 2 \qquad \text{and} \qquad L_1 : \begin{cases} x = & 0 & +t \\ x = & 0 & +3t \\ z = & 1 & \end{cases}$$

20. Graph the following (in $\mathbb{R}^3$).

    (a) $y = 3$

    (b) $z = 0$

    (c) $y = 3x$

    (d) $z = x^2 + y^2$

    (e) $z^2 = x^2 + y^2$

    (f) $z^4 = x^2 + y^2$

    (g) $z^2 = x^2 - y^2$

    (h) $x^2 + y^2 + (z/4)^2 = 1$

21. Let $\mathbf{r}(t) = \langle 3\cos(t), 5\sin(t)\rangle$.

    (a) What is the velocity, acceleration of this particle

    (b) Compute $\mathbf{T}(t)$ and $\mathbf{N}(t)$.

(c) Graph $\mathbf{r}(t)$.

(d) Graph $\mathbf{v}(1)$, and $\mathbf{a}(1)$ coming out from the point $\mathbf{r}(1)$. What does this tell you?

22. Let $\mathbf{r}(t) = \langle 3t^2, t\rangle$.

    (a) What is the velocity, acceleration of this particle

    (b) Compute $\mathbf{T}(t)$ and $\mathbf{N}(t)$.

    (c) Graph $\mathbf{r}(t)$.

    (d) Graph $\mathbf{v}(1)$, and $\mathbf{a}(1)$ coming out from the point $\mathbf{r}(1)$. What does this tell you?

23. Let $\mathbf{r}(t) = \langle 3t, 5t, t^2\rangle$. Compute $\mathbf{T}(t)$, $\mathbf{N}(t)$

24. Let $\mathbf{r}(t) = \langle 3\cos(2t), 5\sin(2t), 4\cos(2t)\rangle$. Compute $\mathbf{T}(t)$, $\mathbf{N}(t)$, and Compute the angle between

25. For the equation $4 = x^2 + y^2$, graph the traces at $z = -1, 0, 1, 2, 3$ and at $x = 0$. Then graph the equation.

26. For the equation $z^2 = x^2 + y^2$, graph the traces at $z = -1, 0, 1, 2, 3$ and at $x = 0$. Then graph the equation.

27. For the equation $z = x^2 + y^2$, graph the traces at $z = -1, 0, 1, 2, 3$ and at $x = 0$. Then graph the equation.

28. Compute

    (a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2+1}{x^2+y^2+2}$

    (b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{e^{x^2+y^2}}{x^2+y^2}$

    (c) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2+xy+y^2}{x^2+y^2}$

29. Let $f(x, y) = x^2y - y^2$. Compute the following.

    (a) $f_x(x, y)$, $f_y(x, y)$, $f_{xy}(x, y)$, $f_{xx}(x, y)$ and $f_{yy}(x, y)$.

    (b) Let $\nabla = \langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}\rangle$. Compute
    $$\nabla f(x, y).$$

    (c) Let $\nabla = \langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}\rangle$. Compute
    $$\nabla \cdot \langle x^3 + y^2, x^2\sin(x)\rangle.$$

30. Let $f(x, y, z) = e^{3x}y^2z^3$. Compute the following.

(a) $f_{yzy}(x, y, z)$.

(b) Let $\nabla = \langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \rangle$. Compute

$$\nabla f(x, y, z).$$

(c) Compute

$$\nabla \times \langle x^3 + y^2, x^2 \sin(x), z^3 \rangle.$$