

$$(1.) \begin{cases} 2x + 4y = -2 \\ 6x + y = 16 \end{cases} \quad \left[\begin{array}{cc|c} 2 & 4 & -2 \\ 6 & 1 & 16 \end{array} \right] \xrightarrow{\frac{1}{2}R_1} \left[\begin{array}{cc|c} 1 & 2 & -1 \\ 6 & 1 & 16 \end{array} \right]$$

$$\begin{array}{l} R_1 \\ R_2 - 6R_1 \end{array} \left[\begin{array}{cc|c} 1 & 2 & -1 \\ 0 & -11 & 22 \end{array} \right] \xrightarrow{\begin{array}{l} R_1 \\ -\frac{1}{11}R_2 \end{array}} \left[\begin{array}{cc|c} 1 & 2 & -1 \\ 0 & 1 & -2 \end{array} \right] \xrightarrow{\begin{array}{l} R_1 - 2R_2 \\ R_2 \end{array}} \left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & -2 \end{array} \right]$$

$$x = 3, y = -2$$

$$(2.) \quad M = \begin{matrix} & C & S \\ \begin{matrix} C \\ S \end{matrix} & \begin{bmatrix} .30 & .20 \\ .20 & .50 \end{bmatrix} \end{matrix} \quad I - M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} .30 & .20 \\ .20 & .50 \end{bmatrix} = \begin{bmatrix} .70 & -.20 \\ -.20 & .50 \end{bmatrix}$$

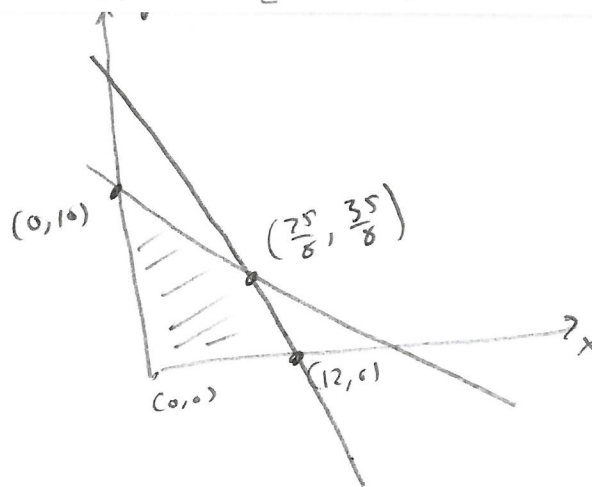
$$X = (I - M)^{-1} D = \begin{bmatrix} .7 & -.2 \\ -.2 & .5 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 10 \\ 20 \end{bmatrix} = \frac{1}{.35 - .04} \begin{bmatrix} .5 & .2 \\ .2 & .7 \end{bmatrix} \begin{bmatrix} 10 \\ 20 \end{bmatrix}$$

$$= \frac{1}{.31} \begin{bmatrix} .5 & .2 \\ .2 & .7 \end{bmatrix} \begin{bmatrix} 10 \\ 20 \end{bmatrix} = \begin{bmatrix} \frac{50}{31} & \frac{20}{31} \\ \frac{20}{31} & \frac{70}{31} \end{bmatrix} \begin{bmatrix} 10 \\ 20 \end{bmatrix} = \begin{bmatrix} 900/31 \\ 1600/31 \end{bmatrix} = \begin{bmatrix} 29.0 \\ 51.6 \end{bmatrix}$$

$$(3.) \quad 5x + 3y \leq 60$$

$$3x + 5y \leq 50$$

$$P = 4x + 3y$$



$$P(0, 0) = 0$$

$$P(0, 10) = 30$$

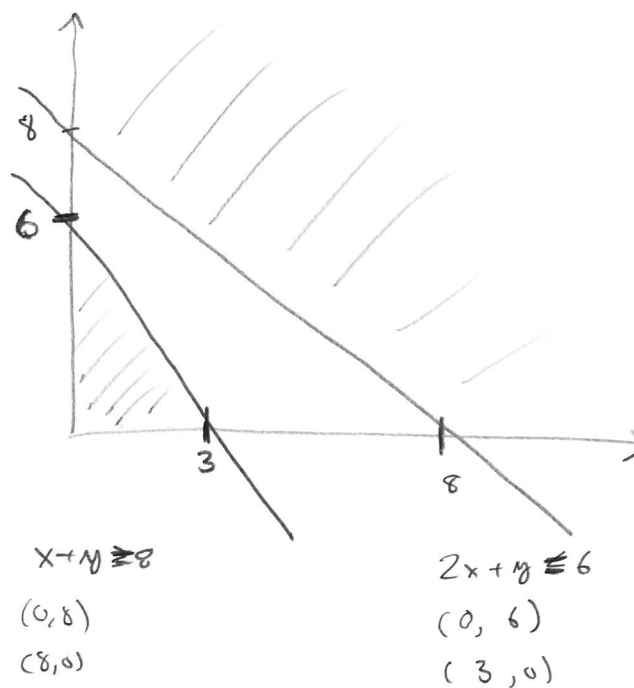
$$P(12, 0) = 48$$

$$P\left(\frac{7.5}{8}, \frac{3.5}{8}\right) = 4\left(\frac{7.5}{8}\right) + 3\left(\frac{3.5}{8}\right)$$

$$P\left(\frac{7.5}{8}, \frac{3.5}{8}\right) = \frac{300 + 105}{8} = \frac{405}{8} \quad \text{maximum}$$

$$(4.) \quad \begin{aligned} 2x + y &\leq 6 \\ x + y &\geq 8 \end{aligned}$$

No feasible region



(5.) Simplex method

$$\begin{aligned} 2x_1 + x_2 &\leq 9 \\ x_1 + x_2 &\leq 6 \\ x_1 + 2x_2 &\leq 10 \end{aligned}$$

$$P = 15x_1 + 20x_2$$

$$\left[\begin{array}{cccccc|c} 2 & 1 & 0 & 0 & 0 & 0 & 9 \\ 1 & 1 & 0 & 1 & 0 & 0 & 6 \\ 1 & 2 & 0 & 0 & 1 & 0 & 10 \\ -15 & -20 & 0 & 0 & 0 & 1 & 0 \end{array} \right] \quad \begin{aligned} 9/1 &= 9 \\ 6/1 &= 6 \\ 10/2 &= 5 \text{ smallest} \end{aligned}$$

← pivot negative

$$\frac{1}{2}R_3 \left[\begin{array}{cccccc|c} 2 & 1 & 0 & 0 & 0 & 0 & 9 \\ 1 & 1 & 0 & 1 & 0 & 0 & 6 \\ 1/2 & 1 & 0 & 0 & 1/2 & 0 & 5 \\ -15 & -20 & 0 & 0 & 0 & 1 & 0 \end{array} \right]$$

$$\begin{aligned} R_1 - R_3 & \quad 3/2 & 0 & 1 & 0 & -1/2 & 0 & 4 & 4/(3/2) = 8/3 \\ R_2 - R_3 & \quad 1/2 & 0 & 0 & 1 & -1/2 & 0 & 1 & 1/(1/2) = 2 \text{ smaller} \\ R_3 & \quad 1/2 & 1 & 0 & 0 & 1/2 & 0 & 5 \\ R_4 + 20R_3 & \quad -5 & 0 & 0 & 0 & 10 & 1 & 100 \end{aligned}$$

$$\begin{array}{l} R_1 \\ 2R_2 \\ R_3 \\ R_4 \end{array} \left[\begin{array}{cc|cc|cc|c} 3/2 & 0 & 1 & 0 & -1/2 & 0 & 4 \\ \boxed{1} & 0 & 0 & 2 & -1 & 0 & 2 \\ 1/2 & 1 & 0 & 0 & 1/2 & 0 & 5 \\ -5 & 0 & 0 & 0 & 10 & 1 & 100 \end{array} \right]$$

Pivot

$$\begin{array}{l} R_1 - \frac{3}{2}R_2 \\ R_2 \\ R_3 - \frac{1}{2}R_2 \\ R_4 + 5R_2 \end{array} \left[\begin{array}{cc|cc|cc|c} 0 & 0 & 1 & -3 & 1 & 0 & 1 \\ 1 & 0 & 0 & 2 & -1 & 0 & 2 \\ 0 & 1 & 0 & -1 & 1 & 0 & 4 \\ 0 & 0 & 0 & 10 & 5 & 1 & 110 \end{array} \right]$$

$\leftarrow x_1 = 2$
 $\leftarrow x_2 = 4$
 $\leftarrow P = 110$

$$x_1 = 2, x_2 = 4, P = 110$$

$$\begin{aligned} (6.) \quad & 10x_1 + 6x_2 \leq 120 \\ & 6x_1 + 10x_2 \leq 100 \end{aligned}$$

$$P = 4x_1 + 3x_2$$

$$\left[\begin{array}{cc|cc|cc|c} \boxed{10} & 6 & 1 & 0 & 0 & 120 \\ 6 & 10 & 0 & 1 & 0 & 100 \\ -4 & -3 & 0 & 0 & 1 & 0 \end{array} \right]$$

$120/10 = 12 \leftarrow \text{smallest}$
 $100/6 = 16.67$

$$\begin{array}{l} R_1 \\ \frac{1}{6.4} R_2 \\ R_3 \end{array} \left[\begin{array}{cc|cc|cc|c} 1 & .6 & .1 & 0 & 0 & 12 \\ 0 & 1 & -.6/6.4 & 10/6.4 & 0 & 28/6.4 \\ 0 & -.6 & .4 & 0 & 1 & 48 \end{array} \right]$$

$$\begin{array}{l} R_1 - .6 R_2 \\ R_2 \\ R_3 + 4 R_1 \end{array} \left[\begin{array}{cc|cc|cc|c} 1 & 0 & .156 & -.938 & 0 & 9.375 \\ 0 & 1 & -.6/6.4 & 10/6.4 & 0 & \boxed{28/6.4} \\ 0 & 0 & .34375 & .938 & 1 & 50.625 \end{array} \right]$$

$$\begin{array}{l} \frac{1}{10} R_1 \\ R_2 \\ R_3 \end{array} \left[\begin{array}{cc|cc|cc|c} 1 & .6 & .1 & 0 & 0 & 12 \\ 6 & 10 & 0 & 1 & 0 & 100 \\ -4 & -3 & 0 & 0 & 1 & 0 \end{array} \right]$$

$$\begin{array}{l} R_2 - 6R_1 \\ R_3 + 4R_1 \end{array} \left[\begin{array}{cc|cc|cc|c} 1 & .6 & .1 & 0 & 0 & 12 \\ 0 & 6.4 & -.6 & 1 & 0 & 28 \\ 0 & -.6 & .4 & 0 & 1 & 48 \end{array} \right]$$

$28/6.4$ smallest

Pivot
Largest Negative

$$\left[x_1 = \frac{75}{8} = 9.375, x_2 = \frac{35}{8} = 4.375 \right]$$

$P = 50.625$

$$7. \quad {}_5P_3 = \frac{5!}{(5-3)!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2!} = \underline{60}$$

$\hookrightarrow = 2!$

$$8. \quad 5^3 = \underline{125}$$

$$9. \quad {}_{14}C_5 = \frac{14!}{(14-5)! 5!} = \frac{14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9!}{9! \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \underline{2002}$$

$$10. \quad {}_5P_5 = \frac{5!}{(5-5)!} = \frac{5!}{0!} = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = \underline{120}$$

Recall $0! = 1$

$$11. \quad n(\text{Off}) = 20 \quad n(D) = 20 \quad n(U) = 60 \quad n((\text{Off} \cup D)^c) = 25$$

$\uparrow \quad \uparrow$
NOT ON LAST YEAR'S TEAM

Solve with
formulas or
using Venn
Diagram

$$\begin{aligned} n(O \cup D) &= n(U) - n((O \cup D)^c) \\ &= 60 - 25 = 35 \end{aligned}$$

$$n(O \cup D) = n(O) + n(D) - n(O \cap D)$$

$$35 = 20 + 20 - n(O \cap D)$$

So $\boxed{n(\text{off} \cap D) = 5}$

(12)

$$(a) \quad n(F) = 19 + 66 = 85$$

$$(b) \quad n(A) = 19 + 14 = 33$$

$$(c) \quad n(A \cup B)^c = 66$$