Show all work and no calculators allowed.

Name: ______________________________

1. Compute the following limits if they exist. If not show why.

   (a) $\displaystyle \lim_{(x,y)\to(0,0)} \frac{\sin(x^2+y^2)}{x^2+y^2}$

   (b) $\displaystyle \lim_{(x,y)\to(0,0)} \frac{x^2+xy+y^2}{x^2+y^2}$

2. Find and classify extremma for $f(x, y) = x^3 + y^2 - 3x - 4y + 2$.

3. Find and classify extremma for $f(x, y, z) = x^2 + y^2 + z^2$ subject to $x - 2y + z = 4$.

4. $\iint_R 4yx\, dA$ over the region R defined in the $xy$-plane as between the graphs of $y = x^2$ and $y = 2x$.

5. $\iint_R \sin(x^2 + y^2)\, dA$ over the region inside the circle $x^2 + y^2 = 4$ and outside of the circle $x^2 + y^2 = 1$ in the third quadrant.

6. $\iint_R \sqrt{3x-y}\,\sin(x+y)\,dA$ over the region defined the lines $y = 3x - 1$, $y = 3x - 4$, $y = -x + 1$ and $y = -x + 2$.

7. $\oint_C \langle x, ye^{x2} \rangle \cdot d\mathbf{r}$ over the region inside the triangle defined by the points $(0, 0)$ to $(2, 6)$ to $(2, -2)$ and back to $(0, 0)$. Notice the path is clockwise.

**Second Derivative Test**

$$D(x,y) = f_{xx}(x,y)f_{yy}(x,y) - (f_{xy}(x,y))^2$$

1. If $D(x,y) > 0$ and $f_{xx}(x,y) < 0$ then the function has a Maximum.
2. If $D(x,y) > 0$ and $f_{xx}(x,y) > 0$ then the function has a Minimum.
3. If $D(x,y) < 0$ then the function has a Saddle Point.
4. If $D(x,y) = 0$ then the test is inconclusive.