MATH 3330 Practice Test 1

1 Parametric Equations

1. Convert to parametric
   (a) $y = x + 3$
   (b) $y = x^2 + 3$
   (c) $y^2 + x = 3$
   (d) $y^2 + x^2 = 3$
   (e) $\frac{y^2}{4} + \frac{x^2}{9} = 1$
2. Convert to rectangular
   (a) $x = t - 3$, $y = t + 3$
   (b) $x = t^2$, $y = t^3$
   (c) $x = 3\cos(t)$, $y = 3\sin(t)$
   (d) $x = \cos(t)$, $y = 3\sin(t)$
3. Graph (without converting to rectangular).
   (a) $x = t - 1$, $y = 2t + 3$
   (b) $x = t^2$, $y = t - 1$
   (c) $x = 3\cos(t)$, $y = 3\sin(t)$
   (d) $x = t\cos(t)$, $y = t\sin(t)$
   (e) $x = t - 1$, $y = 2t + 3$ for $0 \leq t \leq 2$. Label the points where $t = 0, t = 1, t = 2$.
4. Find the equation of the tangent line to the function

$$x = t^4 + t^2 + 2t,\ y = t^3 + t + 1$$

   given parametrically at $t = 1$.
5. Find the area under the curve

$$x = t^4 + t^2,\ y = t^3 + 1$$

   given parametrically from $t = 0$ to $t = 2$.

6. Find the arc length for the function

$$x = 3t + 1,\ y = t^{3/2}$$

given parametrically from $t = 0$ to $t = 2$.

7. Find the arc length for the function

$$x = 4\cos(t),\ y = 4\sin(t)$$

given parametrically from $t = 0$ to $t = \pi$.

8. Find the arc length for

$$x = \cos(t^2),\ y = \sin(t^2)$$

given parametrically from $t = 0$ to $t = \pi$.

2 Polar Coordinates

9. Graph the following

   (a) $r = 3\cos(\theta)$

   (b) $r = 3\cos(\theta) + 2$

   (c) $r = 4\sin(\theta)$

   (d) $r = \theta$

10. Covert the following to rectangular coordinates.

   (a) $r = 3\cos(\theta)$

   (b) $r = 3\cos(\theta) + 2$

   (c) $r = 4\sin(\theta)$

   (d) $r = \theta$

   (e) $r = 2$

   (f) $r = 2\cos(\theta) - 3$

   (g) $r = 2\cos(2\theta)$

   (h) $r = 2\sin(2\theta)$

11. Covert the following given in rectangular coordinates into polar coordinates.

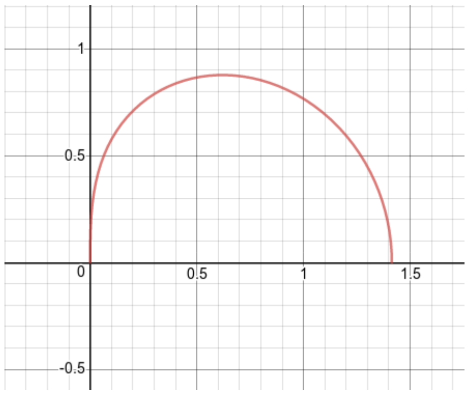

Figure 1: I have graphed $r^2 = 2\cos(\theta)$ from $\theta = 0$ to $\theta = \pi/2$.

(a) $y = x^2$

(b) $x^2 + y^2 = 4$

(c) $y = \sqrt{3}x$

(d) $(x^2 + y^2)^2 + 4x(x^2 + y^2) - 4y^2 = 0$

12. Graph the each region described below. Also compute the area of that region.

(a) Inside $r = 2$ from $\theta = 0$ to $\theta = \pi/4$.

(b) Inside $r^2 = 2\cos(\theta)$ from $\theta = 0$ to $\theta = \pi/4$.

(c) Inside $r = 2\cos(\theta)$ from $\theta = 0$ to $\theta = \pi/4$.

(d) Inside $r = 2\cos(\theta)$ and inside $r = 1$.

(e) Inside $r = 2\cos(\theta)$ and outside $r = 1$.

(f) Inside $r = 2\cos(3\theta)$ (a three petaled rose).

3 Vectors

13. Define the three points $P(1, 0, 2)$, $Q(-1, 1, 0)$ and $R(1, 2, 2)$ and the vectors $\mathbf{v} = \langle 1, 2, 2\rangle$, $\mathbf{w} = \langle 1, 2, 3\rangle$ and $\mathbf{u} = \langle 0, 0, 1\rangle$.

(a) Find a unit vector parallel to $\vec{PQ}$.

(b) Find the equation of a line that contains P and R. Find both the parametric and the vector equation.

(c) Find the equation of a plane that contains P and R. Find all three forms from class: the parametric, the vector equation and the normal equation.

(d) Find the angle between the vectors **v** and **w**.

(e) Find the area of the parallelogram formed by vectors **v** and **w**.

(f) Find the volume of the parallelepiped formed by vectors **v**, **w** and **u**.

14. Graph the parallelogram formed by the following four points: $A(0, 0, 0)$, $B(2, 3, 0)$ $C(1, 4, -1)$, and $D(3, 7, -1)$. Find the area of the parallelogram.

15. Consider the three lines

$$L_1 : x = 2t,\ y = 3 - 4t,\ z = 2 + 6t$$
$$L_2 : x = -t,\ y = 3 + 2t,\ z = 2 - 3t$$
$$L_3 : x = 2,\ y = 2t - 5,\ z = 6 + t$$

and the two planes

$$P_1 : x - y = 14$$
$$P_2 : x + y - 4z = 0$$

(a) Are the lines L_1 and L_2 parallel? Why or why not?

(b) Are the lines L_1 and L_3 parallel? Why or why not?

(c) Find the angle between the line L_2 and the plane P_1.

(d) Find the angle between the plane P_1 and the plane P_2.

(e) Where do the lines L_3 and L_2 intersect (if they do)? Show this.

(f) Where do the lines L_3 and L_1 intersect (if they do)? Show this.

(g) Where does the line L_1 intersect plane P_1 (if they do)? Show this.

(h) Find a parametric form for the plane P_1.

(i) Find an equation of the plane formed by the intersection of lines L_1 and L_3.

16. Graph the following. Graph the level curves and the entire function.

(a) $z^2 = x^2 + y^2$

(b) $z = x^2 + y^2$

(c) $z^3 = x^2 + y^2$

(d) $z = x^2 - y^2$

(e) $x^2 + y^2 + z^2 = 2$

17. Graph the equation given in cylindrial coordinates.

$$r = 3cos(\theta)$$

18. Graph the equation given in cylindrial coordinates.

$$r = zcos(\theta)$$

19. Graph the equations given in spherical coordinates.

(a)

$$\phi = \pi/4$$

(b)

$$\rho = 2$$

(c)

$$\theta = \pi/4$$

4 Vector Valued Functions

20. Let $\mathbf{r}(t) = \langle t^2 - t, t^3 - 3t^2 + 3\rangle$

(a) Find the position, velocity and acceleration of the particle at time $t = 2$.

(b) Graph the position, velocity and acceleration appropriately.

21. Let $\mathbf{r}(t) = \langle \sin(e^{-t}), \cos(e^{-t})\rangle$

(a) Find the speed function

$$\frac{ds}{dt} = \|\mathbf{r}'(t)\|$$

(b) Compute the arc length from $t = 0$ to $t = 1$.

(c) Compute the arc length from $t = 0$ to $t = \infty$.

(d) What is the graph of $\mathbf{r}(t)$?

22. Let $\mathbf{r}(t) = \langle e^{-t}\sin(t), e^{-t}\cos(t)\rangle$

    (a) Find the speed function

    $$\frac{ds}{dt} = \|\mathbf{r}'(t)\|$$

    (b) Compute the arc length from $t = 0$ to $t = 1$.

    (c) Compute the arc length from $t = 0$ to $t = \infty$.

    (d) What is the graph of $\mathbf{r}(t)$?

23. Compute the following limits if they exist. If not show why.

    (a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^3 + y^3}{x^2 + y^2 + 1}$

    (b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{1 - \cos(x^2 + y^2)}{x^2 + y^2}$

    (c) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 + xy + y^2}{x^2 + y^2}$

24. Let $f(x, y) = x^2 + y^2$. Consider the points $P(0, 2)$ and $Q(1, 2)$.

    (a) Graph the contour plot. Include $z = -1, 0, 1, 2, 3, 4$.

    (b) ~~Compute the $\nabla f(x, y)$.~~

    (c) ~~Compute the $\nabla f(P)$ and $\nabla f(Q)$. Also compute their norms.~~

    (d) ~~Graph $\nabla f(P)$ and $\nabla f(Q)$ with initial points P and Q respectively.~~

5 Two easy proofs

25. Prove

$$\|\mathbf{v} + \mathbf{w}\|^2 = \|\mathbf{v}\|^2 + \|\mathbf{w}\|^2 + 2\mathbf{v}\cdot\mathbf{w}$$

26. Prove

$$\|\mathbf{v}\|^2\|\mathbf{w}\|^2 = (\mathbf{v}\cdot\mathbf{w})^2 + \|\mathbf{v}\times\mathbf{w}\|^2$$