1 Integration

1. $\int \dfrac{1}{x \ln(x)} dx$ u-sub

2. $\int \dfrac{1}{x(\ln(x))^2} dx$ u-sub

3. $\int \tan(x) dx$

4. $\int \sec(x) dx$

5. $\int \dfrac{1}{\sqrt{x^2 - 4}} dx$ hint: trig sub

6. $\int \dfrac{2x + 2}{x^2 + 2x} dx$

7. $\int \dfrac{2x^2 + x + 1}{x^3 + x} dx$

8. $\int \dfrac{2x^2 + 3x + 1}{x(x + 1)^2} dx$

9. $\int_1^\infty \dfrac{1}{x^2} dx$

10. $\int_1^\infty \dfrac{1}{x} dx$

11. $\int_1^\infty \dfrac{1}{\sqrt{x}} dx$

2 Limits

12. $\lim\limits_{n \to \infty} (1 + \dfrac{1}{n})^n$

13. $\lim\limits_{x \to 0^+} (x)^x$

14. $\lim\limits_{x \to 0^+} (1+x)^{\frac{1}{x}}$

15. $\lim\limits_{x \to 0^+} (1+5x)^{\frac{1}{2x}}$

3 Series

16. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n} - \frac{1}{n+1}$

17. $\displaystyle\sum_{n=1}^{\infty} \ln\left(\frac{n+1}{n}\right)$

18. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{3^n}$

19. $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{2^n}$

20. $\displaystyle\sum_{n=7}^{\infty} e^{-n}$

21. $\displaystyle\sum_{n=7}^{\infty} \frac{1}{n \ln(n)}$

22. $\displaystyle\sum_{n=1}^{\infty} \left(1 - \frac{1}{n}\right)^n$

23. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+1}{3n^2+1}$

24. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$

25. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$

26. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

27. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+1}{n^3+1}$

28. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+1}{n^4+1}$

29. $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n+5}}{n^2+1}$

30. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n+n^2}{3^n+n^3}$

31. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{1}{n}$

32. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{1}{n^2}$

33. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n}}$

34. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n!}$

35. $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{n!}$

36. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{e^n}$

37. $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{n^2}$

38. $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{n!}$

39. $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^n}$

40. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n^2 + 1}{3n^2 + 1} \right)^n$

41. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{3n^2 + 1}{2n^2 + 1} \right)^n$

42. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{2n^2}{3n^2 + 1} \right)^n$

43. $\displaystyle\sum_{n=1}^{\infty} \left(1 - \frac{1}{n} \right)^{n^2}$

4 Power Series and Taylor Series

44. Find the interval of convergence for the the following power series.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2} x^n$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} x^n$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2^n} (x - 1)^n$

   (d) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2 2^n} (x - 1)^n$

45. Find the Taylor series for the given functions. Show your work and the nth term. Use the definition.

   (a) $f(x) = e^{3x}$ centered at $x = 0$.
   (b) $f(x) = \sin(x)$ centered at $x = 0$.
   (c) $f(x) = \sin(x)$ centered at $x = \pi/2$.
   (d) $f(x) = \ln(x)$ centered at $x = 1$.

46. Find the Taylor series for the given functions. Show your work and the nth term. Use the a known series. You are expected to know the Taylor series for e^x, $\sin(x)$, $\cos(x)$, and $\frac{1}{1-x}$.

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(a) $f(x) = e^{3x}$ centered at $x = 0$.

(b) $f(x) = \sin(x^2) - x^2$ centered at $x = 0$.

(c) $f(x) = \frac{\sin(x^2) - x^2}{x^6}$ centered at $x = 0$.

(d) $f(x) = xe^{x^2}$ centered at $x = 0$.