Math 2320 - Final Exam Review

The final exam will include topics from Test 1 and Test 2. For example there will be an integral of each type we have learned. I would study the reviews from Test 1, Test 2 and this review.

1 Definition of the integral

1. Compute the definite integral using the definition.

$$\int_0^3 (2x^2 + 3)\, dx$$

2 Techniques of Integration

2. u-sub

   (a) $\int x^3(x^4 + 1)^{3/4}\, dx$

   (b) $\int x \sec(x^2 + 1) \tan(x^2 + 1)\, dx$

   (c) $\int \frac{x^3 + 1}{x^4 + x - 7}\, dx$

3. By Parts

   (a) $\int xe^{2x}\, dx$

   (b) $\int x^2 \cos(3x)\, dx$

   (c) $\int \ln(x)\, dx$

   (d) $\int \arctan(x)\, dx$ Hint $u = \arctan(x)$, and $dv = 1dx$.

4. Trigonometric Integrals

   (a) $\int \sin^3(x + 1)\, dx$

   (b) $\int \sin^2(3x)\, dx$

(c) $\int \sin^{3/4}(2x)\cos(2x)\,dx$

5. Trigonometric Substitution

   (a) $\int \frac{1}{\sqrt{x^2-1}}\,dx$

   (b) $\int \sqrt{4-9x^2}\,dx$

   (c) $\int \frac{1}{(1+x^2)^{3/2}}\,dx$

   (d) $\int \frac{1}{x^2(x^2-1)^{3/2}}\,dx$

6. Partial Fractions

   (a) $\int \frac{1}{x^2(x-1)}\,dx$

   (b) $\int \frac{x^3+1}{x(x^2+1)}\,dx$

   (c) $\int \frac{x^2+x+1}{x^3-3x^2-10x}\,dx$

3 Taylor Series & Power Series

7. Find the Interval of convergence and the radius of convergence for the following power series. Make certain to test the endpoints of the interval.

   (a) $\sum_{n=1}^{\infty} \frac{1}{n3^n} x^n$

   (b) $\sum_{n=1}^{\infty} \frac{1}{n^2 3^n} x^n$

   (c) $\sum_{n=1}^{\infty} \frac{1}{n!} x^n$

   (d) $\sum_{n=1}^{\infty} \frac{2^n}{n!} x^n$

   (e) $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!} x^n$

8. Find the Taylor series from the formula. Find the first four non-zero terms and the n^{th} term at the indicated point.

   (a) $f(x) = e^{2x}$ at $x_0 = 0$

   (b) $f(x) = e^{2x}$ at $x_0 = 1$

(c) $f(x) = x^4$ at $x_0 = 1$

(d) $f(x) = \cos(2x)$ at $x_0 = \pi$

9. Find the Taylor series from a known series. Find the first four non-zero terms and the n^{th} term at the indicated point.

   (a) $f(x) = e^{2x}$ at $x_0 = 0$

   (b) $f(x) = e^{x^2} - 1$ at $x_0 = 0$

   (c) $f(x) = \ln(1 + x^3)$ at $x_0 = 0$

   (d) $f(x) = \frac{\cos(x) - 1 - \frac{1}{2}x^2}{x^4}$ at $x_0 = 0$

10. Use a Taylor series representation to evaluate the following limits

    (a) $\lim_{x \to 0} \frac{e^x - 1 - x}{x^2}$

    (b) $\lim_{x \to 0} \frac{\sin(x^2) - x^2}{x^4}$

4 Parametric Equations

11. Find the parametric equation for the following equations given in rectangular coordinates.

    (a) $y = x^2$

    (b) $y = 3x - 1$

    (c) $y^2 + y = 2x + 2$

12. Find equation in rectangular coordinates for the following equations given in the parametrically.

    (a) $x = 3t$ and $y = 2t - 1$

    (b) $x = 3t$ and $y = 2t^2 - 1$

    (c) $x = 4\cos(2t)$ and $y = 4\sin(2t)$

    (d) $x = \cos(t)$ and $y = 3\sin(t)$

13. Graph the following parametric equations

    (a) $x = 3t$ and $y = 2t - 1$

    (b) $x = 3t$ and $y = 2t^2 - 1$

    (c) $x = \cos(t)$ and $y = 3\sin(t)$

    (d) $x = t\cos(t)$ and $y = t\sin(t)$

5 Polar Coordinates

14. Graph the following given in polar equations.

    (a) $r = 3$

    (b) $r = 4\sin(\theta)$

    (c) $r = \sin(2\theta)$

    (d) $r = 1 + 2\sin(\theta)$

15. Convert the following given in polar equations into rectangular.

    (a) $r = 3$

    (b) $r = 4\sin(\theta)$

    (c) $r = \sin(2\theta)$ Hint use a trigonometric identity $\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$.

    (d) $r = 1 + 2\sin(\theta)$