

Math 3160 - Test 1 Review

Be certain to **know** the quizzes and . . .

1 Systems of Linear Equations

1. Solve the following systems of linear equations using row reduction.

$$(a) \begin{cases} x_1 - 2x_2 & & -6x_5 & = 0 \\ & x_2 & +x_3 & +6x_4 & = 5 \\ & 2x_2 & & +6x_4 & +x_5 & = 4 \\ & x_2 & -x_3 & & +x_5 & = -1 \end{cases}$$

$$(b) \begin{cases} 2x_1 - 2x_2 + 4x_3 & = 2 \\ & x_3 & = 0 \\ x_1 + x_2 + 2x_3 & = 0 \end{cases}$$

$$(c) \begin{cases} 2x_1 - 2x_2 + 4x_3 & = 2 \\ -x_1 - x_2 + 3x_3 & = 2 \\ x_1 - 3x_2 + 7x_3 & = 2 \end{cases}$$

2. Solve the following systems of linear equations by setting up problem as a matrix problem and by finding an inverse matrix.

$$(a) \begin{cases} 2x_1 - 2x_2 + 4x_3 & = 2 \\ & -x_2 + 3x_3 & = 2 \\ & -3x_2 + 7x_3 & = 2 \end{cases}$$

$$(b) \begin{cases} 2x_1 - 2y & = 2 \\ -x_1 - 3y & = 2 \end{cases}$$

2 Matrices, Determinants and Cramer's Rule

$$3. \text{ Let } A = \begin{bmatrix} 2 & -2 & 0 \\ -1 & 4 & 1 \\ 0 & 4 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 & -2 & 0 & 0 \\ 2 & -2 & 1 & 2 \\ 2 & -2 & 0 & 3 \\ 2 & -2 & 5 & 4 \end{bmatrix}, C = \begin{bmatrix} 2 & -2 & 0 & -2 & 0 \\ 0 & -2 & 0 & -2 & 0 \\ 0 & 0 & 3 & -2 & 0 \\ 0 & 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix}$$

$$\text{and } D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

- (a) Find the determinant of the matrices A, B, C and D
 - (b) Compute D^3 and D^{-1} .
 - (c) Compute $C^T C$. What kind of matrix is $C^T C$?
4. Solve the following equations for X assuming any matrix has an inverse. Let A, B, C, X be $n \times n$ matrices and let $\mathbf{u}$ be an $n \times 1$ vector.
- (a) $AX = BX - A$
 - (b) $AX = 2X - A$
 - (c) $A\mathbf{u} = 2\mathbf{u} + B$
5. ~~Assume $\det(A) = 11$.~~
- ~~(a) Switch R1 and R2 in the matrix A to get matrix B, what is $\det(B)$?~~
 - ~~(b) Replace R1 with $R1 - 4R2$ to get matrix B, what is $\det(B)$?~~
 - ~~(c) Multiply R1 by 4 to get matrix B, what is $\det(B)$?~~
6. Solve the following using Cramer's rules.
- (a)
$$\begin{cases} 2x_1 & -2x_2 & +4x_3 & = 2 \\ & -x_2 & +3x_3 & = 2 \\ & -3x_2 & +7x_3 & = 2 \end{cases}$$
 - (b)
$$\begin{cases} 2x_1 & -2y & = 2 \\ -x_1 & -3y & = 2 \end{cases}$$

3 Vectors

7. Let $\mathbf{v} = (1, 3, 4)$ and $\mathbf{w} = (1, -1, 0)$ be vectors in $\mathbb{R}^3$. and let $P(1, 1, 1)$ and $Q(0, -4, 0)$ be two points in $\mathbb{R}^3$.
- (a) Find a vector that is parallel to $\mathbf{v}$ and unit.
 - (b) Compute $\|2\mathbf{v} - \mathbf{w}\|$.
 - (c) Compute the angle between $\mathbf{v}$ and $\mathbf{w}$.
 - (d) Find the equation of a plane containing P and with normal vector $\mathbf{v}$.
8. Find parametric equations for the line (in $\mathbb{R}^3$) so that
- (a) the line contains the point $P(0, 1, 2)$ and $Q(1, 2, 3)$.

- (b) the line contains the point $P(-5, 1, 3)$ and is parallel to the vector $(1, 2, 3)$.
9. Find equation for the plane (in $\mathbb{R}^3$) so that
- (a) the plane contains the point $P(2, 2, 2)$, $Q(1, 2, 3)$ and $R(1, -1, 0)$.
 - (b) the plane contains the origin and is perpendicular to the vector $(1, 2, 3)$.
10. Define the planes P_1 and P_2 as follows:

$$P_1 : x - 2y + z = 12$$

$$P_2 : 3x - 3y + z = 4$$

- (a) What are the two normal vectors for the above planes.
 - (b) Find the angle between the two above planes.
 - (c) Find two points one on each of the above planes.
 - (d) Find set of all points that lay in both planes.
11. Let $\mathbf{v} = (1, 3, 4)$ and $\mathbf{w} = (1, -1, 0)$ be vectors in $\mathbb{R}^3$.
- (a) Compute $\mathbf{v} \times \mathbf{w}$.
 - (b) Compute $\mathbf{w} \times \mathbf{v}$.
 - (c) Compute $\mathbf{w} \times \hat{\mathbf{i}}$.
 - (d) Compute $(\sin(\theta), \cos(\theta), 1) \times (\cos(\theta), -\sin(\theta), 0)$.
 - (e) What is the area contained within the parallelogram formed by the vectors $\mathbf{v}$ and $\mathbf{w}$.
 - (f) What is the area contained within the triangle defined by the three vertices $P(1, 0, 1)$, $P(-2, 0, 0)$ and the origin?
 - (g) what is the volume of the parallelepiped formed by the three vectors $\mathbf{v}$, $\hat{\mathbf{i}}$ and $\mathbf{w}$.