Math 3160 - Quiz 6

Name:

1. Let $B = \{(1, 2, 1), (0, 1, 2), (0, -1, 0)\}.$

- (a) Is B a basis for \mathbb{R}^3
- (b) Write the vector (1,0,-1) relative to the basis B.
- (c) Write the vector (a, b, c) relative to the basis B.
- (d) Find the change of basis matrix from the standard basis to the basis B. (we called it $P_{\text{STANDARD}\to B}$ in class).
- 2. For the following system of linear equations.

$$2x_1 -2x_2 +4x_3 -6x_5 = 2$$

 $x_3 +6x_4 = 0$

- (a) Find the solution set.
- (b) Find a basis for the solution set.
- (c) What is the dimension of that solution set?
- 3. For the following subspace of P_3

$$W = \{a + bx + cx^2 + dx^3 : a = -c \text{ and } b = c + d\}$$

- (a) Find a basis for W.
- (b) What is the dimension of that solution set?
- 4. Find a a basis and dimension for the following.

(a)
$$span((1,-2,1),(1,1,1),(-2,3,-2))$$

(b)
$$\{(x, y, z, w) \in \mathbb{R}^4 : x + y - 2z = 0 \text{ and } 3y - z - w = 0\}$$

- 5. Let $B_1 = \{(-1,1),(2,3)\}, B_2 = \{(1,-1),(1,1)\}$ and let B be the standard unit basis for \mathbb{R}^2 .
 - (a) Find the change of basis matrices for $P_{B_1 \to B_2}$ and $P_{B_1 \to B_2}$.
 - (b) Find the coordinates of the point (4,6) (given in the standard basis) relative to the bases B_1 and B_2 .
 - (c) Find the change of basis matrices for $P_{B\to B_2}$ and $P_{B_2\to B}$.
 - (d) Find the coordinates of the point (2, -4) (given in the standard basis) relative to the bases B and B_2 . Graph this point the two separate coordinate axes B and B_2 .

6. Diagonalize the following matrices if possible. If not possible state why.

$$A = \begin{bmatrix} 1 & 3 \\ 1 & -1 \end{bmatrix}, B = \begin{bmatrix} 6 & 3 & -8 \\ 0 & -2 & 0 \\ 1 & 0 & -3 \end{bmatrix}$$
 and

$$C = \left[\begin{array}{rrr} 4 & 0 & -1 \\ 0 & 3 & 0 \\ 1 & 0 & 2 \end{array} \right]$$