

Math 6250 Presentation 4

Name: _____

1. Show the following are monotone or not. State whether they are monotone increasing, monotone decreasing or not monotone. And prove it.
 - (a) $a_n = \frac{1}{n}$.
 - (b) Defined recursively as $a_1 = 1$ and $a_{n+1} = 1 + \frac{1}{a_n}$.
 - (c) Defined recursively as $a_1 = 1$ and $a_{n+1} = 1 + \frac{a_n}{a_n+1}$.
 - (d) $a_n = \frac{-1}{n}$.
 - (e) What do the sequences from 1b and 1c have to do with a well known sequence?
2. Prove the Monotone convergence Theorem: That is
If (a_n) is a bounded and monotone sequence then (a_n) converges.
3. Use the Monotone Convergence Theorem to show that: the sequence defined as $a_1 = 1$ and $a_{n+1} = 1 + \frac{a_n}{a_n+1}$ converges.
4. Prove with $\varepsilon - N$ proof that $a_n = \frac{2n+1}{3n+5}$ converges.
5. Prove with $\varepsilon - N$ proof that the sequence defined below is not convergent.

$$a_n = \sum_{j=1}^n \frac{1}{j}$$

So $a_1 = \sum_{j=1}^1 \frac{1}{j} = \frac{1}{1} = 1$, $a_2 = \sum_{j=1}^2 \frac{1}{j} = \frac{1}{1} + \frac{1}{2} = \frac{3}{2}$ and $a_3 = \sum_{j=1}^3 \frac{1}{j} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} = \frac{11}{6}$.