

Math 6250 Quiz 1

Name: _____

1. Notice the following

$$1 + 2 + 3 + 2 + 1 = 9$$

$$1 + 2 + 3 + 4 + 3 + 2 + 1 = 16$$

$$1 + 2 + 3 + 4 + 5 + 4 + 3 + 2 + 1 = 25$$

Make a conjecture as to a rule for all $n \in \mathbb{N}$ and prove using induction. Note this can be shown geometrically. See if you can prove this by just drawing a few squares.

2. Define the following relation on \mathbb{Z}

$$a \sim b \Leftrightarrow 5 \mid a - b$$

- (a) Show \sim is an equivalence relation.
- (b) Prove the usual addition on \mathbb{Z} is well defined for this equivalence relation.
- (c) Prove the usual exponentiation on \mathbb{Z} is **not** well defined for this equivalence relation.

3. Here we use an equivalence relation on \mathbb{N} to define the integers. Let $a, b \in \mathbb{N}$. Define the following relation.

$$(a, b) \mathcal{R}(a', b') \Leftrightarrow a + b' = a' + b$$

And define multiplication as

$$(a, b) + (c, d) = (a + c, b + d)$$

$$(a, b) \cdot (c, d) = (ac + bd, ad + bc)$$

- (a) Show \mathcal{R} is an equivalence relation.
 - (b) Show addition is well defined. That is, Show
If $(a, b) \mathcal{R}(a', b')$ and $(c, d) \mathcal{R}(c', d')$ then $(a, b) + (c, d) \mathcal{R}(a', b') + (c', d')$.
 - (c) Show multiplication is well defined.
4. Write the definition for the equivalence relation on \mathbb{Z} to define \mathbb{Q} . Show it is an equivalence relation and show that multiplication is well defined.
5. Show if $x^2 = 5$ then x is not a rational number.