

Math 3160 - Extra Quiz

Name: _____

For the following show all work clearly.

1. Find the eigenvalues and bases of eigenspaces (the eigenvectors) for

$$A = \begin{bmatrix} 4 & -5 \\ 1 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}.$$

2. Rewrite the following in terms of $\langle u, v \rangle$ and $\|u\|$ and $\|v\|$. Make certain to justify each step.

$$\langle 2v - 4u, v + 3u \rangle$$

3. Prove Theroem 6.1.1 parts a and c from your book.
4. Answer the questions for the standard inner product defined on \mathcal{P}_2 .

- (a) $\|x^2\|$
- (b) $d(x^2, 1 - x - x^2)$
- (c) the angle θ between x^2 and $1 - x - x^2$.

5. Answer the questions for the inner product defined on \mathcal{P}_2 .

$$\langle f, g \rangle = \int_{-1}^1 fg \, dx$$

- (a) $\|x^2\|$
- (b) $d(x^2, 1 - x - x^2)$
- (c) the angle θ between x^2 and $1 - x - x^2$.

6. Answer the questions for the evaluation inner product defined on \mathcal{P}_2 using the points $x_0 = 1, x_2 = 2$, and $x_3 = -1$.

- (a) $\|x^2\|$
- (b) $d(x^2, 1 - x - x^2)$
- (c) the angle θ between x^2 and $1 - x - x^2$.

7. Sketch the unit circle for $\langle (x_1, y_1), (x_2, y_2) \rangle = 3x_1x_2 + 3y_1y_2$ on \mathbb{R}^2 .

8. Use GS Orthogonaliztion to find an ON basis for the following.

- (a) $\{(1, 3), (-1, 2)\}$ on the inner product space defined in problem 7.
 - (b) $\{1, 2x^2, x - 1\}$ on the inner product space defined in problem 4.
 - (c) $\{1, 2x^2, x - 1\}$ on the inner product space defined in problem 5.
 - (d) $\{1, 2x^2, x - 1\}$ on the inner product space defined in problem 6.
9. Find W^\perp for the following subspaces W with the defined inner products.
- (a) $W = \{(x, y, z) \in \mathbb{R}^3 \mid 3x + 2y = z\}$ with the Euclidean inner product (the usual dot product).
 - (b) $W = \{p(x) \in \mathcal{P}_2 \mid p'(x) = 0\}$ with the integral inner product defined in Problem 5.