

## Math 6250: Test 1

Name: \_\_\_\_\_

1. Do **one** of the following:

- (a) Show if  $x^2 = 5$  then  $x$  is not a Rational number.
- (b) Define the relation on  $\mathbb{Z}$  by

$$aRb \iff 3|a^2 - b^2.$$

Prove  $R$  is an equivalence relation.

2. Prove

If  $f$  is injective and  $g$  is injective then  $f \circ g$  is injective.

3. Prove

If  $(a_n)$  and  $(b_n)$  are convergent then  $(a_n + b_n)$  is convergent.

4. Prove the  $\lim_{n \rightarrow \infty} \frac{2n^2+1}{5n^2+2} = \frac{2}{5}$  using the  $\varepsilon - N$  definition.

5. Prove  $(a_n)$  converges given the sequence below

$$a_1 = 3 \text{ and } a_{n+1} = \sqrt{a_n + 1} \text{ for all } n \in \mathbb{N}$$

And find its limit.

6. Solve  $x^4 = i$

7. Find and prove a trigonometric identity for  $\cos(4\theta)$  using Euler's formula.