



Department of Mathematics, Computer & Information Science

**Advanced Linear Algebra
MA 4160 • SYLLABUS SPRING 2016**

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Office Hours: M 3:50PM – 4:45PM, TR 5:20PM – 7:00PM

TEXTBOOK: Linear Algebra Done Right, by Sheldon Axler. Springer; 3rd ed. 2015.
ISBN-10: 3319110799.

PREREQUISITES: Grade of C or higher in Linear Algebra MA 3160 and grade of C or higher in Discrete Mathematics MA 3030.

COURSE DESCRIPTION: This course is a proof based course. We extend the ideas from MA3160: Linear Algebra to more abstract structures and study linear operators in detail. We will study inner product spaces, orthonormal bases and normed spaces, duality on infinite dimensional vector spaces, spectral theorems and the decomposition of operators. This course in a sense is backwards from the traditional treatment of linear algebra. The last idea introduced is the determinant (a MA 3160: Linear Algebra topic). We will introduce the determinant from scratch and define it in geometric or combinatorial terms.

COURSE OBJECTIVES: As in all proof based courses, the student should be able to write complete and clear mathematical proofs. Also the successful student should be able to compute a dual space, compute minimal polynomials, use the Canonical Forms. And identify an inner product space and prove that the space is an inner product space. Prove the basic properties. Compute spectra and the decompositions of Linear Operators.

And a successful student should know what the determinant is.

COURSE EVALUATION & GRADING: Your grade for the course will be based on your homework/quiz performance (15%), two tests (50%) and a comprehensive final exam (35%).

A = [94, 100]	B+ = [87, 89]	C+ = [77, 79]	D+ = [67, 69]	F = [0, 59]
A- = [90, 93]	B = [84, 86]	C = [74, 76]	D = [64, 66]	
	B- = [80, 83]	C- = [70, 73]	D- = [60, 63]	

CALCULATOR: No calculator is allowed nor needed.

OFFICE OF SERVICES FOR STUDENTS WITH DISABILITIES: SUNY/Old Westbury is committed to assuring that all students have equal access to learning and extracurricular activities on campus. If you have, or suspect you may have a physical, psychological, medical or learning disability that may impact how you function academically and/or your access to activities on campus, please contact Dr. Lisa Whitten, Director of the Office of Services for Students with Disabilities (OSSD). She will work with you to determine which accommodations you need, and provide you with documentation for your professors. The OSSD is located in the NAB, Room 2064. OSSD services are free and confidential. In addition, we hire qualified note takers at \$100.00 for the semester if you are enrolled in the course, and \$9.00 an hour if you are not. You can reach Dr. Whitten at 516-876-3009 or whittenl@oldwestbury.edu.

RESPECT: No cell phones in class and no texting.

FINAL EXAM: Will be held May 19, 2016 in our regular classroom at the regular class time.

Topics Covered

1. Vector Spaces
 - 1A \mathbb{R}^n and \mathbb{C}^n
 - 1B Definition of Vector Space
 - 1C Subspaces
2. Finite-Dimensional Vector Spaces
 - 2A Span and Linear Independence
 - 2B Bases
 - 2C Dimension
3. Linear Maps
 - 3.A The Vector Space of Linear Maps
 - 3.B Null Spaces and Ranges
 - Null Space and Injectivity
 - Range and Surjectivity
 - Fundamental Theorem of Linear Maps
 - 3.C Matrices
 - Representing a Linear Map by a Matrix
 - Addition and Scalar Multiplication of Matrices
 - Matrix Multiplication
 - 3.D Invertibility and Isomorphic Vector Spaces
 - Invertible Linear Maps
 - Isomorphic Vector Spaces
 - Linear Maps Thought of as Matrix Multiplication
 - Operators
 - 3.F Duality
 - The Dual Space and the Dual Map
 - The Null Space and Range of the Dual of a Linear Map
 - The Matrix of the Dual of a Linear Map
 - The Rank of a Matrix
4. Polynomials
 - Complex Conjugate and Absolute Value
 - Uniqueness of Coefficients for Polynomials
 - The Division Algorithm for Polynomials
 - Zeros of Polynomials
 - Factorization of Polynomials over \mathbb{C}
 - Factorization of Polynomials over \mathbb{R}
5. Eigenvalues, Eigenvectors, and Invariant Subspaces
 - 5.A Invariant Subspaces
 - Eigenvalues and Eigenvectors
 - Restriction and Quotient Operators
 - 5.B Eigenvectors and Upper-Triangular Matrices
 - Polynomials Applied to Operators
 - Existence of Eigenvalues
 - Upper-Triangular Matrices
 - 5.C Eigenspaces and Diagonal Matrices
6. Inner Product Spaces
 - 6.A Inner Products and Norms
 - Inner Products
 - Norms
 - 6.B Orthonormal BasesLinear Functionals on Inner Product Spaces
 - 6.C Orthogonal Complements and Minimization Problems
 - Orthogonal Complements
 - Minimization Problems
7. Operators on Inner Product Spaces
 - 7.A Self-Adjoint and Normal Operators

- Adjoint
- Self-Adjoint Operators
- Normal Operators
- 7.B The Spectral Theorem
 - The Complex Spectral Theorem
 - The Real Spectral Theorem
- 7.C Positive Operators and Isometries
 - Positive Operators
 - Isometries
- 7.D Polar Decomposition and Singular Value Decomposition
 - Polar Decomposition
 - Singular Value Decomposition
- 8. Operators on Complex Vector Spaces
 - 8.A Generalized Eigenvectors and Nilpotent Operators
 - Null Spaces of Powers of an Operator
 - Generalized Eigenvectors
 - Nilpotent Operators
 - 8.B Decomposition of an Operator
 - Description of Operators on Complex Vector Spaces
 - Multiplicity of an Eigenvalue
 - Block Diagonal Matrices
 - Square Roots
 - 8.C Characteristic and Minimal Polynomials
 - The Cayley–Hamilton Theorem
 - The Minimal Polynomial
 - 8.D Jordan Form
- 9. Operators on Real Vector Spaces
 - 9.A Complexification
 - Complexification of a Vector Space
 - Complexification of an Operator
 - The Minimal Polynomial of the Complexification
 - Eigenvalues of the Complexification
 - Characteristic Polynomial of the Complexification
 - 9.B Operators on Real Inner Product Spaces
 - Normal Operators on Real Inner Product Spaces
 - Isometries on Real Inner Product Spaces
- 10. Trace and Determinant
 - 10.A Trace
 - Change of Basis
 - 10.B Determinant
 - Determinant of an Operator
 - Determinant of a Matrix
 - The Sign of the Determinant
 - Volume

We will cover Chapters 1 through 10 in this book. Chapters 4 and 9 we will only see quick overview and in Chapter 3 we will skip quotient spaces.