

MATH 2320 Practice Test 2

1 Techniques of integration

1. Compute the following integrals using by parts.

- (a) $\int x e^{2x} dx$
- (b) $\int x^2 e^{3x} dx$
- (c) $\int x^3 e^{3x^2} dx$
- (d) $\int x \sin(2x) dx$
- (e) $\int x^2 \sin(2x) dx$
- (f) $\int \ln(x) dx$
- (g) $\int x \ln(x) dx$
- (h) $\int x^3 \ln(x) dx$
- (i) $\int \arctan(2x) dx$

2. Compute the following trig integrals

- (a) $\int \sin(5x) \cos(5x) dx$
- (b) $\int \sin(3x) \cos^3(3x) dx$
- (c) $\int \sin(7x) \cos^{1/2}(7x) dx$
- (d) $\int \sin^2(3x) dx$
- (e) $\int \sin^2(2x) \cos^2(2x) dx$
- (f) $\int \sec(x) dx$
- (g) $\int \sec^2(x) dx$
- (h) $\int \tan(x) dx$
- (i) $\int \tan^2(x) dx$
- (j) $\int \csc(x) dx$
- (k) $\int \cot(x) dx$
- (l) $\int \sec^3(x) dx$

3. Compute the following using trig substitutions.

- (a) $\int x^3 \sqrt{9 - x^2} dx$
- (b) $\int \frac{x^3}{\sqrt{4 - x^2}} dx$

- (c) $\int \frac{x^2}{4+x^2} dx$
- (d) $\int \frac{1}{\sqrt{x^2-4}} dx$
- (e) $\int \frac{x^2}{\sqrt{x^2-4}} dx$
4. Compute the following using partial fraction decomposition.
- (a) $\int \frac{1}{4-x^2} dx$
- (b) $\int \frac{2x^2-3}{4x-x^3} dx$
- (c) $\int \frac{3x^2-2}{4x+x^3} dx$
- (d) $\int \frac{3x^3-2}{4x+x^3} dx$
5. Compute the following improper integrals
- (a) $\int_0^1 \frac{1}{x^2} dx$
- (b) $\int_1^\infty \frac{1}{x^2} dx$
- (c) $\int_0^1 \frac{1}{\sqrt{x}} dx$
- (d) $\int_0^\infty \frac{1}{\sqrt{x}} dx$
- (e) $\int_0^\infty \sin(x) dx$
- (f) $\int_{-\infty}^\infty \frac{1}{1+x^2} dx$
- (g) $\int_0^\infty x e^{-x^2} dx$
- (h) $\int_0^\infty x e^{-x^2} dx$

2 Limits of sequences

6. Compute the following limits
- (a) $\lim_{n \rightarrow \infty} \frac{1}{n^2}$
- (b) $\lim_{n \rightarrow \infty} \frac{3n^2-2n+1}{5n^2+8}$
- (c) $\lim_{n \rightarrow \infty} \frac{\sqrt{n}(3n^2-2n+1)}{5n^2+8}$
- (d) $\lim_{n \rightarrow \infty} \frac{\sqrt{n}(2n+1)}{\sqrt{5n^3+8}}$
- (e) $\lim_{x \rightarrow 0} (1-2x)^{1/x}$
- (f) $\lim_{x \rightarrow 0} (1+5x^2)^{1/x}$

- (g) $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$
- (h) $\lim_{n \rightarrow \infty} \left(1 + \frac{3}{n}\right)^n$
- (i) $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n^2}\right)^{n^2}$
- (j) $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n^2}\right)^n$
- (k) $\lim_{n \rightarrow \infty} \frac{1}{n!}$
- (l) $\lim_{n \rightarrow \infty} \frac{1}{n^n}$
- (m) $\lim_{n \rightarrow \infty} \frac{n!}{2(n!)}$
- (n) $\lim_{n \rightarrow \infty} \frac{n!}{(2n!)}$
- (o) $\lim_{n \rightarrow \infty} \frac{(n!)^2}{(2n!)}$
- (p) $\lim_{n \rightarrow \infty} \frac{e^n}{n^2}$
- (q) $\lim_{n \rightarrow \infty} \frac{e^n}{2^n}$
- (r) $\lim_{n \rightarrow \infty} \frac{\ln(n)}{n^2}$
- (s) $\lim_{n \rightarrow \infty} \frac{e^n}{n!}$
- (t) $\lim_{n \rightarrow \infty} \frac{n^{23}+10}{e^n}$

3 Telescoping

- (a) $\sum_{k=1}^{\infty} \frac{1}{k} - \frac{1}{k+2}$
- (b) $\sum_{k=1}^{\infty} \sqrt{k+1} - \sqrt{k}$
- (c) $\sum_{k=1}^{\infty} \frac{1}{k(k+1)}$

4 Geometric Series

- (a) $\sum_{k=1}^{\infty} 3^{-k}$
- (b) $\sum_{k=1}^{\infty} \frac{1}{3^{-k}}$

$$(c) \sum_{k=17}^{\infty} 11 \cdot 3^{-k}$$

$$(d) \sum_{k=1}^{\infty} \frac{2^k + 3^k}{4^k}$$

$$(e) \sum_{k=1}^{\infty} \frac{4^k}{2^k + 3^k}$$

$$(f) 9 + -3 + 1 + -\frac{1}{3} + \frac{1}{9} + \dots$$

5 Integral Test

$$\text{i. } \sum_{k=1}^{\infty} e^{-k}$$

$$\text{ii. } \sum_{k=1}^{\infty} \frac{1}{1 + k^2}$$

$$\text{iii. } \sum_{k=17}^{\infty} \frac{1}{k \ln(k)}$$

$$\text{iv. } \sum_{k=17}^{\infty} \frac{1}{k \ln^2(k)}$$

6 P-Series Test

$$\text{i. } \sum_{k=1}^{\infty} \frac{1}{\sqrt{k}}$$

$$\text{ii. } \sum_{k=1}^{\infty} \frac{1}{k^2}$$

$$\text{iii. } \sum_{k=1}^{\infty} k^3$$

$$\text{iv. } \sum_{k=1}^{\infty} k^{-3}$$

$$\text{v. } \sum_{k=1}^{\infty} \frac{k + k^2}{k^3}$$

$$\text{vi. } \sum_{k=1}^{\infty} \frac{k + k^2}{k^4}$$

7 Ratio Test

$$\text{i. } \sum_{k=1}^{\infty} \frac{k}{2^k}$$

$$\text{ii. } \sum_{k=1}^{\infty} \frac{1}{k!}$$

$$\text{iii. } \sum_{k=1}^{\infty} \frac{2^k}{k!}$$

$$\text{iv. } \sum_{k=1}^{\infty} \frac{k!}{k^k}$$

$$\text{v. } \sum_{k=1}^{\infty} \frac{(k!)^2}{k^k}$$

$$\text{vi. } \sum_{k=1}^{\infty} \frac{(k!)^2}{(2k)!}$$

$$\text{vii. } \sum_{k=1}^{\infty} \frac{(2k)!}{k^k}$$