Fall 2016 MA 5320: Test 1

Name: _____

1. Prove If the sup(A) exists then $\sup(A) = -\inf(-A)$.

2. Prove the following:
   Let A be a nonempty subset of $\mathbb{R}$ and let $\alpha = \sup(A)$. If $\varepsilon > 0$ then there exists an $x \in A$ so that
   $$\alpha - \varepsilon < x \leq \alpha.$$

3. Prove either
   (a) If $a_n \to a$ and $b_n \to b$ then $\lim_{n\to\infty} a_n b_n = ab$, or
   (b) If (a_n) is convergent then (a_n) is bounded.

4. $\lim_{n\to\infty} \frac{n+1}{3n+5} = \frac{1}{3}$

5. For the following questions use
   $$a_1 = 6, \text{ and } a_n = \sqrt{3 + a_{n-1}}$$
   (a) Prove (a_n) is monotone.
   (b) Prove (a_n) is bounded.
   (c) Use the MCT (and state the MCT) to prove (a_n) converges.
   (d) What is the limit?

6. Find the limit and prove it for the following using the $\varepsilon - \delta$ definition for the limit
   $$\lim_{x\to -2} x^2 + 2x$$

7. Define $f : \mathbb{R} \to \mathbb{R}$ by $f(x) = x^2 - 1$. Prove f is continuous from the definition.

8. Let $f : \mathbb{R} \to \mathbb{R}$ satisfy $|f(x) - f(y)| \leq |x - y|^p$ where $p \geq 1$ for all $x, y \in \mathbb{R}$. Prove f is uniformly continuous.