

**MATH 5320 Test 2**

**Name:** \_\_\_\_\_

1. Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be given by  $f(x) = x^2 + 2x - 3$ . Prove using the  $\varepsilon - \delta$  definition that  $f$  is continuous.
2. Let  $f : \mathbb{R} \rightarrow \{0, 1\}$  be continuous. Prove  $f(x)$  is constant.
3. Prove if  $f(x)$  and  $g(x)$  are differentiable at  $x = c$  then  $[f(c)g(c)]' = f'(c)g(c) + f(c)g'(c)$ .
4. Let  $f : [0, \infty) \rightarrow \mathbb{R}$  be thrice differentiable. Assume  $f(0) = f'(0) = f''(0) = 0$  and assume  $f'''(x) > 8$  for all  $x \in [0, \infty)$ . Prove that  $f(x) > x^3$  for all  $x \in [0, \infty)$ .
5. Let  $f(x) = 3x + 1$ ,  $[a, b] = [1, 4]$  and let  $\mathcal{P} = \{1, 2, 3.5, 3.7, 4\}$  be a Partition for  $[1, 4]$ . Let  $\mathcal{S} = \{1, 2.2, 3.6, 4\}$  be a sampling. Compute  $RS(f, \mathcal{P}, \mathcal{S})$
6. Define  $f(x) = \begin{cases} \frac{x^3 - 5x^4}{|x|} - 5x + 4 & : x \neq 0 \\ 4 & : x = 0 \end{cases}$ .
  - (a) Show  $f$  is continuous at  $x = 0$ .
  - (b) Compute  $f'(0)$  if it exists. If it doesn't exist show why.
7. Let  $f : [0, 1] \rightarrow \mathbb{R}$  be defined by  $f(x) = \begin{cases} x & : x \in \mathbb{Q} \\ -x & : x \in \mathbb{R} \setminus \mathbb{Q} \end{cases}$ .
  - (a) Graph  $f(x)$ .
  - (b) Let  $\mathcal{P} = \{0, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1\}$  Find a sampling,  $\mathcal{S}_1$  of  $\mathcal{P}$  so that
$$RS(f, \mathcal{P}, \mathcal{S}_1) > 1/2.$$
  - (c) Let  $\mathcal{P} = \{0, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1\}$  Find a sampling,  $\mathcal{S}_2$  of  $\mathcal{P}$  so that
$$RS(f, \mathcal{P}, \mathcal{S}_2) < -1/2.$$