Math 3520 - Quiz 7

Name:_____

Type up your answers in complete English.

1. Prove or disprove G1, G2, G3 and G4 for the following algebraic structure $(S, \otimes)$ where $S = \mathbb{Z}$ and $a \otimes b = 2ab - a - b + 1$. Hint there is compute $a \otimes 1$ and $1 \otimes a$.

2. Write down the addition table for $(\mathbb{Z}_6, +)$ and the multiplication tables for $(\mathbb{Z}_7^*, *)$. Compare the two tables. make a correspondence between the tables that preserves the operation.

3. As we did in class define a group to be the symmetries of the hexagon. Describe your group.

4. Define $S = \{\sigma \in S_4 : \sigma^2 = e\}$. Show S along with composition is a group. Prove it is an abelian group.

5. State and prove the cancellation laws.

6. Prove $A = \{2^n : n \in \mathbb{Z}\}$ is a subgroup of $(\mathbb{Q}^*, \cdot)$.

7. Prove $(\mathbb{Z}_6, +)$ and $(\mathbb{Z}_7^*, *)$ are isomorphic. You must define an isomorphism, ϕ and prove ϕ is an isomorphism.