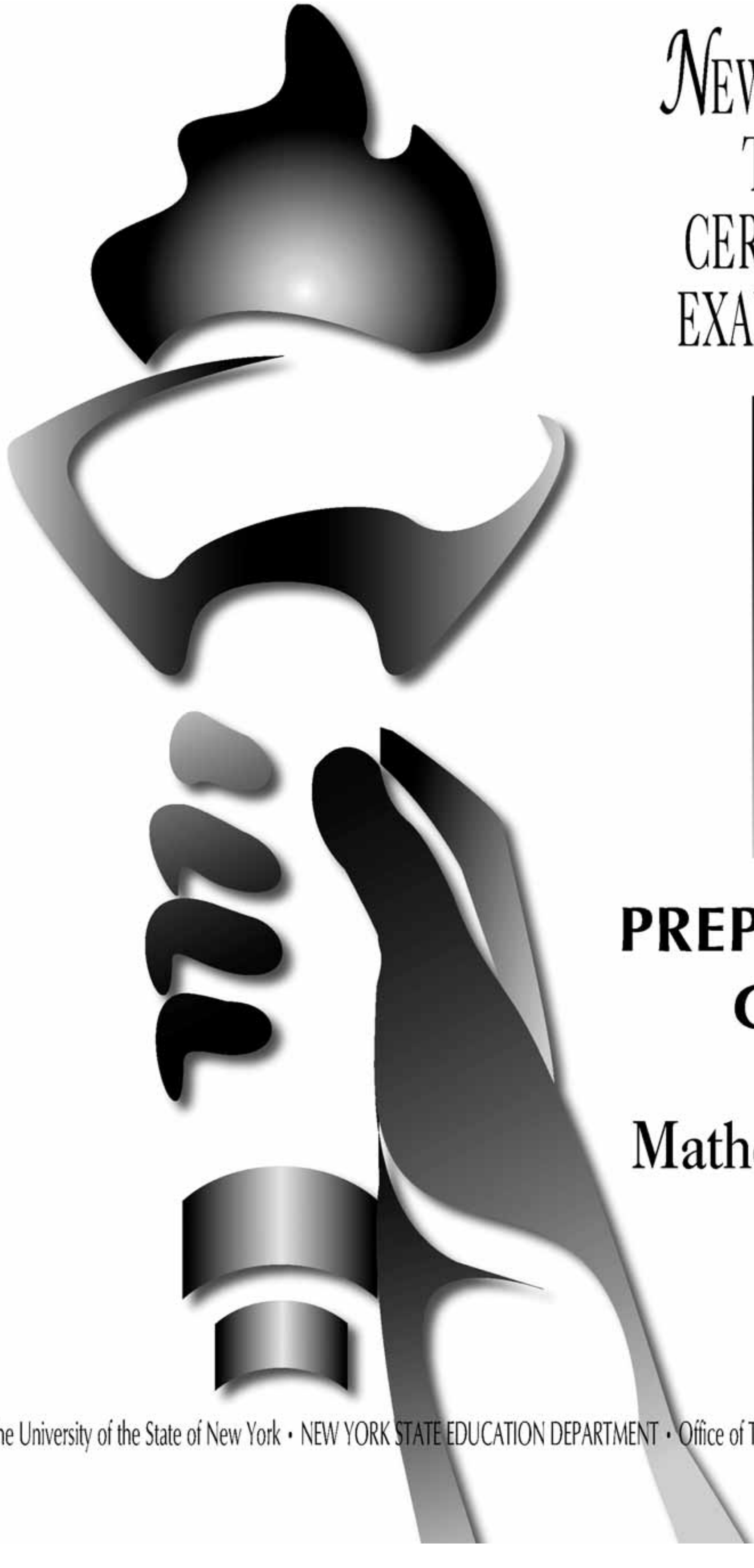

NEW YORK STATE TEACHER CERTIFICATION EXAMINATIONS™

NYSTCE®

PREPARATION GUIDE

Mathematics CST (04)

The University of the State of New York • NEW YORK STATE EDUCATION DEPARTMENT • Office of Teaching Initiatives, Albany, New York 12234

NY-SG-FLD004-02

NEW YORK STATE TEACHER CERTIFICATION EXAMINATIONS™

PREPARATION GUIDE
Mathematics CST (04)

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INTRODUCTION

Purpose of This Preparation Guide

This preparation guide is designed to help familiarize candidates with the content and format of a test for the New York State Teacher Certification Examinations (NYSTCE®) program. Education faculty and administrators at teacher preparation institutions may also find the information in this guide useful as they discuss the test with candidates.

The knowledge and skills assessed by the test are acquired throughout the academic career of a candidate. A primary means of preparing for the test is the collegiate preparation of the candidate.

This preparation guide illustrates some of the types of questions that appear on a test; however, the set of sample questions provided in this preparation guide does not necessarily define the content or difficulty of an entire actual test. All test components (e.g., directions, question content and formats) may differ from those presented here. The NYSTCE program is subject to change at the sole discretion of the New York State Education Department.

Organization of This Preparation Guide

Contained in the beginning sections of this preparation guide are general information about the NYSTCE program and how the tests were developed, a description of the organization of test content, and strategies for taking the test.

Following these general information sections, specific information about the test described in this guide is presented. The test objectives appear on the pages following the test-specific overview. The objectives define the content of the test.

Next, information about the multiple-choice section of the test is presented, including sample test directions. Sample multiple-choice questions are also presented, with the correct responses indicated and explanations of why the responses are correct.

Following the sample multiple-choice questions, a description of the written assignment section of the test is provided, including sample directions. A sample written assignment is presented next, followed by a sample strong response to the assignment and an evaluation of that response.

For Further Information

If you have questions after reading this preparation guide, you may wish to consult the NYSTCE Registration Bulletin. You can view or print the registration bulletin online at **www.nystce.nesinc.com**.

GENERAL INFORMATION ABOUT THE NYSTCE

How Were the NYSTCE Tests Developed?

The New York State Teacher Certification Examinations are criterion referenced and objective based. A criterion-referenced test is designed to measure a candidate's knowledge and skills in relation to an established standard rather than in relation to the performance of other candidates. The explicit purpose of these tests is to help identify for certification those candidates who have demonstrated the appropriate level of knowledge and skills that are important for performing the responsibilities of a teacher in New York State public schools.

Each test is designed to measure areas of knowledge called subareas. Within each subarea, statements of important knowledge and skills, called objectives, define the content of the test. The test objectives were developed for the New York State Teacher Certification Examinations in conjunction with committees of New York State educators.

Test questions matched to the objectives were developed using, in part, textbooks; New York State learning standards and curriculum guides; teacher education curricula; and certification standards. The test questions were developed in consultation with committees of New York State teachers, teacher educators, and other content and assessment specialists.

An individual's performance on a test is evaluated against an established standard. The passing score for each test is established by the New York State Commissioner of Education based on the professional judgments and recommendations of New York State teachers. Examinees who do not pass a test may retake it at any of the subsequently scheduled test administrations.

Organization of Content

The content covered by each test is organized into **subareas**. These subareas define the major content domains of the test.

Subareas typically consist of several **objectives**. Objectives provide specific information about the knowledge and skills that are assessed by the test.

Each objective is elaborated on by **focus statements**. The focus statements provide examples of the range, type, and level of content that may appear on the tests.

Test questions are designed to measure specific test objectives. The number of objectives within a given subarea generally determines the number of questions that will address the content of that subarea on the test. In other words, the subareas that consist of more objectives will receive more emphasis on the test and contribute more to a candidate's test score than the subareas that consist of fewer objectives.

The following example, taken from the field of Social Studies, illustrates the relationship of test questions to subareas, objectives, and focus statements.

SOCIAL STUDIES (05)

SUBAREA I—HISTORY

0003 Understand the major political, social, economic, scientific, and cultural developments and turning points that shaped the course of world history from 1500 through 1850.

analyzing the roles, contributions, and diverse perspectives of individuals and groups involved in independence struggles in Latin America

Which of the following was an important goal of nineteenth-century Latin American liberals?

A. establishing governments based on the separation of church and state

B. reducing the influence of competitive individualism in social and economic life

C. creating strong centralized governments

D. making plantation agriculture the foundation of economic development

→ Each multiple-choice question is designed to measure one of the test objectives.

→ The focus statements provide examples of the range, type, and level of content that may appear on the test for questions measuring the objective.

→ The objectives define the knowledge and skills that New York State teachers and teacher educators have determined to be important for teachers to possess.

→ The field is divided into major content subareas. The number of objectives in each subarea may vary, depending on the breadth of content contained within it.

→ This is the name and field number of the test.

TEST-TAKING STRATEGIES

Be On Time.

Arrive at the test center on time so that you are rested and ready to begin the test when instructed to do so.

Follow Directions.

At the beginning of the test session and throughout the test, follow all directions carefully. This includes the oral directions that will be read by the test administrators and any written directions in the test booklet. The test booklet will contain general directions for the test as a whole and specific directions for individual test questions or groups of test questions. If you do not understand something about the directions, do not hesitate to raise your hand and ask your test administrator.

Pace Yourself.

The test schedule is designed to allow sufficient time for completion of the test. Each test session is four hours in length. The tests are designed to allow you to allocate your time within the session as you need. You can spend as much time on any section of the test as you need, and you can complete the sections of the test in any order you desire; however, you will be required to return your materials at the end of the four-hour session.

Since the allocation of your time during the test session is largely yours to determine, planning your own pace for taking the test is very important. Do not spend a lot of time with a test question that you cannot answer promptly; skip that question and move on. If you skip a question, be sure to skip the corresponding row of answer choices on your answer document. Mark the question in your test booklet so that you can return to it later, but be careful to appropriately record on the answer document the answers to the remaining questions.

You may find that you need less time than the four hours allotted in a test session, but you should be prepared to stay for the entire time period. Do not make any other commitments for this time period that may cause you to rush through the test.

Read Carefully.

Read the directions and the questions carefully. Read all response options. Remember that multiple-choice test questions call for the "best answer"; do not choose the first answer that seems reasonable. Read and evaluate all choices to find the best answer. Read the questions closely so that you understand what they ask. For example, it would be a waste of time to perform a long computation when the question calls for an approximation.

Read the test questions, but don't read into them. The questions are designed to be straight-forward, not tricky.

Mark Answers Carefully.

Your answers for all multiple-choice questions will be scored electronically; therefore, the answer you select must be clearly marked and the only answer marked. If you change your mind about an answer, erase the old answer completely. Do not make any stray marks on the answer document; these may be misinterpreted by the scoring machine.

IF YOU SKIP A MULTIPLE-CHOICE QUESTION, BE SURE TO SKIP THE CORRESPONDING ROW OF ANSWER CHOICES ON YOUR ANSWER DOCUMENT.

You may use any available space in the test booklet for notes, but **your answers and your written response must be clearly marked on your answer document. ONLY ANSWERS AND WRITTEN RESPONSES THAT APPEAR ON YOUR ANSWER DOCUMENT WILL BE SCORED.** Answers and written responses in your test booklet will not be scored.

Guessing

As you read through the response options, try to find the best answer. If you cannot quickly find the best answer, try to eliminate as many of the other options as possible. Then guess among the remaining answer choices. Your score on the test is based on the number of test questions that you have answered correctly. There is no penalty for incorrect answers; therefore, it is better to guess than not to respond at all.

Passages or Other Presented Materials

Some test questions are based on passages or other presented materials (e.g., graphs, charts). You may wish to employ some of the following strategies while you are completing these test questions.

One strategy is to read the passage or other presented material thoroughly and carefully and then answer each question, referring to the passage or presented material only as needed. Another strategy is to read the questions first, gaining an idea of what is sought in them, and then read the passage or presented material with the questions in mind. Yet another strategy is to review the passage or presented material to gain an overview of its content, and then answer each question by referring back to the passage or presented material for the specific answer. Any of these strategies may be appropriate for you. You should not answer the questions on the basis of your own opinions but rather on the basis of the information in the passage or presented material.

Check Accuracy.

Use any remaining time at the end of the test session to check the accuracy of your work. Go back to the test questions that gave you difficulty and verify your work on them. Check the answer document, too. Be sure that you have marked your answers accurately and have completely erased changed answers.

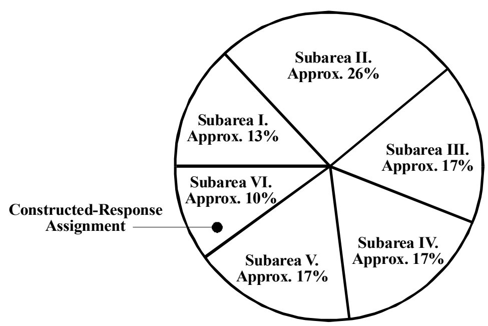
ABOUT THE MATHEMATICS TEST

The purpose of the Mathematics Content Specialty Test (CST) is to assess knowledge and skills in the following six subareas:

Subarea I.	Mathematical Reasoning and Communication
Subarea II.	Algebra
Subarea III.	Trigonometry and Calculus
Subarea IV.	Measurement and Geometry
Subarea V.	Data Analysis, Probability, Statistics, and Discrete Mathematics
Subarea VI.	Algebra: Constructed-Response Assignment

The test objectives presented on the following pages define the content that may be assessed by the Mathematics CST. Each test objective is followed by focus statements that provide examples of the range, type, and level of content that may appear on the test for questions measuring that objective.

The test contains approximately 90 multiple-choice test questions and one constructed-response (written) assignment. The figure below illustrates the approximate percentage of the test corresponding to each subarea.



The section that follows the test objectives presents sample test questions for you to review as part of your preparation for the test. To demonstrate how each objective may be assessed, a sample question is presented for each objective. The correct response and an explanation of why the response is correct follow each question. A sample written assignment is also presented, along with an example of a strong response to the assignment and an evaluation of that response.

The sample questions are designed to illustrate the nature of the test questions; they should not be used as a diagnostic tool to determine your individual strengths and weaknesses.

A section containing mathematical formulas will be provided in the Mathematics test booklet. Sample formulas pages can be found before the sample Mathematics test questions in this guide.

MATHEMATICS TEST OBJECTIVES

Mathematical Reasoning and Communication
Algebra
Trigonometry and Calculus
Measurement and Geometry
Data Analysis, Probability, Statistics, and Discrete Mathematics
Algebra: Constructed-Response Assignment

The New York State mathematics educator has the knowledge and skills necessary to teach effectively in New York State public schools. The mathematics teacher is adept at utilizing the mathematical systems of algebra, geometry, trigonometry, and calculus and can address and solve problems involving data analysis, probability, statistics, and discrete mathematics. The mathematics teacher is able to reason logically and understands the connections between mathematics and other disciplines. Most importantly, the mathematics teacher is able to communicate mathematically and use language skills to explain mathematical concepts and processes, is able to apply mathematics in real-world settings, and is able to solve problems through the integrated use of multiple mathematical skills and concepts.

SUBAREA I—MATHEMATICAL REASONING AND COMMUNICATION

0001 Understand reasoning processes, including inductive and deductive logic and symbolic logic.

For example:

- analyzing mathematical situations by gathering evidence, making conjectures, formulating counterexamples, and constructing and evaluating arguments
- analyzing the nature and purpose of axiomatic systems (including those of the various geometries)
- analyzing and interpreting the truth value of simple and compound statements (e.g., negations, disjunctions, conditionals) in truth tables and Venn diagrams
- using laws of inference to draw conclusions and to test the validity of conclusions
- applying the principle of mathematical induction to prove theorems

0002 Understand the meaning of mathematical concepts and symbols and how to communicate mathematical ideas in writing.

For example:

- translating among algebraic, graphic, numeric, and written modes of presenting mathematical ideas
- converting between mathematical language, notation, and symbols and standard English language
- deducing the assumptions inherent in a given mathematical statement, expression, or definition
- evaluating the precision or accuracy of a mathematical statement

0003 Understand mathematical modeling and apply multiple mathematical representations to connect mathematical ideas and solve problems.

For example:

- evaluating mathematical models (e.g., graphs, equations, physical and pictorial representations) and analyzing their appropriateness, efficiency, and accuracy in solving a given problem
- analyzing techniques of estimation and identifying situations in which estimation is appropriate
- using estimation to evaluate the reasonableness of a solution to a problem
- representing problem situations using a variety of mathematical models (e.g., algebraic expressions, sequences, diagrams, geometric figures, graphs)
- analyzing the use of software (e.g., simulations, spreadsheets) to model and solve problems

SUBAREA II—ALGEBRA

0004 Understand principles and properties of the set of complex numbers and its subsets.

For example:

- applying principles of number theory (e.g., prime numbers, divisibility) to solve problems
- applying number concepts (e.g., fractions, percents, exponents) to solve problems
- applying knowledge of real numbers to arithmetic and algebraic operations
- justifying the need for the extension of a given number system
- using multiple representations of the complex numbers and their operations (e.g., polar form; algebraic and geometric interpretations of the sum, difference, and product of complex numbers)
- analyzing and applying the properties of vectors, groups, and fields to the complex numbers and its subsets

0005 Understand the principles and properties of patterns and algebraic operations and relations.

For example:

- determining algebraic expressions that best represent patterns among data presented in tables, graphs, and diagrams
- generalizing patterns using explicitly defined and recursively defined functions
- performing and analyzing basic operations on numbers and algebraic expressions
- deriving an algebraic model that best represents a given situation and evaluating the strengths and weaknesses of that model
- applying algebraic concepts of relation and function (e.g., range, domain, inverse) to analyze mathematical relationships
- analyzing the results of transformations (e.g., translations, dilations, reflections, rotations) on the graphs of functions

0006 Understand the properties of linear functions and relations.

For example:

- analyzing a linear equation in terms of slope and intercepts
- determining the linear function that best models a set of data
- solving systems of linear equations and inequalities using a variety of techniques (e.g., algebraic, graphic, matrix)
- applying properties of linear equations and inequalities to model and solve a variety of real-world problems
- using techniques of linear programming to model and solve real-world problems
- determining connections among proportions, linear functions, and constant rates of change

0007 Understand the properties of quadratic and higher-order polynomial functions and relations.

For example:

- analyzing the characteristics of the roots of quadratic and higher-order polynomial functions
- solving systems of quadratic equations or inequalities using a variety of techniques (e.g., factoring, graphing, completing the square, quadratic formula)
- modeling and solving a variety of mathematical and real-world problems involving quadratic functions and relations
- analyzing symbolic, graphic, or tabular representations of a given quadratic or higher-order polynomial relation
- analyzing the results of changing parameters on the graphs of quadratic and higher-order polynomial functions
- applying properties of polynomials to model and solve problems

0008 Understand the properties of rational, radical, and absolute value functions and relations.

For example:

- analyzing the properties of a given function (e.g., range, domain, asymptotes)
- solving problems involving rational, radical, and absolute value functions using various algebraic techniques
- modeling and solving problems graphically using systems of equations and inequalities involving rational, radical, and absolute value relations (including the use of graphing calculators)
- interpreting and analyzing the effects of transformations on the graph of a function
- applying the properties of rational, radical, and absolute value functions and relations to model and solve problems

0009 Understand the properties of exponential and logarithmic functions.

For example:

- analyzing the relationship between logarithmic and exponential functions
- applying the laws of logarithms to manipulate and simplify expressions
- analyzing the graph of a logarithmic or exponential function or relation
- solving problems involving exponential growth and decay
- modeling and solving problems analytically or graphically using exponential and logarithmic relations

SUBAREA III—TRIGONOMETRY AND CALCULUS

0010 Understand principles, properties, and relationships involving trigonometric functions and their associated geometric representations.

For example:

- using degree and radian measures
- analyzing connections and identities among right triangle ratios, trigonometric functions, and the unit circle
- analyzing characteristics of the graph of a trigonometric function (e.g., frequency, period, amplitude, phase shift)
- determining connections between trigonometric functions and power series
- using the complex exponential function to explore properties of trigonometric functions

0011 Apply the principles and techniques of trigonometry to model and solve problems.

For example:

- applying trigonometric functions to solve problems involving length, area, volume, or angle measure (e.g., arcs, angles and sectors associated with a circle, unknown sides and angles of polygons, vectors)
- using circular functions to model periodic phenomena
- solving trigonometric equations using analytic or graphing techniques
- modeling and solving problems involving trigonometric functions

0012 Demonstrate an understanding of the fundamental concepts of calculus.

For example:

- analyzing the concept of limit numerically, algebraically, graphically, and in writing
- interpreting the derivative as the limit of the difference quotient
- interpreting the definite integral as the limit of a Riemann sum
- applying the fundamental theorem of calculus
- applying concepts of derivatives to interpret gradients, tangents, and slopes
- applying the concept of limit to analyze and interpret the properties of functions (e.g., continuity, asymptotes)
- applying the concept of rate of change to interpret statements from science, technology, economics, and other disciplines

0013 Apply the principles and techniques of calculus to model and solve problems.

For example:

- using derivatives to model and solve real-world problems (e.g., rates of change, related rates, optimization)
- applying properties of derivatives to analyze the graphs of functions
- using integration to model and solve problems (e.g., the area under a curve, work, applications of antiderivatives)
- modeling and solving problems involving first order differential equations (e.g., separation of variables, initial value problems)

SUBAREA IV—MEASUREMENT AND GEOMETRY

0014 Understand and apply measurement principles.

For example:

- applying appropriate measurement tools and units and converting within and between measurement systems (e.g., conversion factors, dimensional analysis)
- recording measurements to the appropriate degree of accuracy and analyzing the effect of uncertainty on derived measures
- deriving and applying formulas to find measures such as length, angle, area, and volume for a variety of geometric figures
- using nets and cross sections to determine volume and surface area formulas of three-dimensional figures
- applying the Pythagorean theorem to solve measurement problems

0015 Understand the principles and properties of axiomatic (synthetic) geometries.

For example:

- using the properties of lines and angles (e.g., parallelism, perpendicularity, supplementary angles, vertical angles) to characterize geometric relationships
- using the concepts of similarity and congruence to analyze the properties of geometric figures (e.g., triangle, parallelogram, polygon, circle)
- analyzing procedures used in geometric constructions (e.g., constructing the perpendicular bisector of a given line segment)
- proving theorems using an axiomatic system
- analyzing, comparing, and contrasting the axiomatic structures and properties of various geometries (e.g., Euclidean, non-Euclidean, projective)
- applying geometric principles to analyze three-dimensional figures

0016 Understand the principles and properties of coordinate geometry.

For example:

- applying the principles of distance, midpoint, slope, parallelism, and perpendicularity to characterize coordinate geometric relationships
- using coordinate geometry to prove theorems about geometric figures (e.g., triangle, parallelogram, circle, parabola, hyperbola)
- representing two- and three-dimensional geometric figures in various coordinate systems (e.g., Cartesian, polar)
- analyzing and applying transformations in the coordinate plane
- applying the distance formula to derive the equation of a conic section
- modeling and solving problems using conic sections

0017 Apply mathematical principles and techniques to model and solve problems involving vector and transformational geometries.

For example:

- modeling and solving problems involving vector addition and scalar multiplication (e.g., force)
- applying principles of geometry to model and solve problems involving the composition of transformations (e.g., translations, reflections, dilations, rotations)
- analyzing how transformational geometry and symmetry are used in art and architecture (e.g., tessellations, tilings, frieze patterns, fractals)
- using multiple representations of geometric transformations (e.g., coordinate, matrix, diagrams)
- proving theorems using vector and transformational methods

SUBAREA V—DATA ANALYSIS, PROBABILITY, STATISTICS, AND DISCRETE MATHEMATICS

0018 Understand the principles, properties, and techniques related to sequence, series, summation, and counting strategies and their applications to problem solving.

For example:

- using first- or second-order finite differences to analyze sequences
- modeling and solving problems using the properties of sequences and series (e.g., arithmetic, geometric, Fibonacci)
- solving a variety of problems involving permutations and combinations
- analyzing the relationship between the binomial coefficients and Pascal's triangle

0019 Understand the principles, properties, and techniques of probability and their applications.

For example:

- evaluating the probability of events (e.g., joint, conditional, independent, mutually exclusive)
- interpreting graphic representations of probabilities (e.g., tables, charts, Venn diagrams, tree diagrams, frequency graphs, the normal curve)
- modeling and solving problems involving uncertainty using the techniques of probability (e.g., addition and multiplication rules, Bernoulli experiment)
- using simulations to estimate probabilities and analyzing the relationships between probability and statistics
- solving problems involving random variability and probability distributions (e.g., binomial, normal)

0020 Understand the principles, properties, and techniques of data analysis and statistics.

For example:

- applying measures of central tendency, dispersion, and skewness to summarize and interpret data presented in graphic, tabular, or pictorial form
- evaluating the statistical claims made for a given set of data (e.g., analyzing assumptions made in the sampling, analysis, and testing of statistical hypotheses) and how measures of reliability vary by discipline
- interpreting the outcomes of a given statistical test (e.g., *t*-test, chi-square analysis, correlation, linear regression)
- using graphing calculators to analyze and interpret data from a variety of disciplines (e.g., sciences, social sciences, technology)
- analyzing data using linear, logarithmic, exponential, and power regression models

0021 Understand how techniques of discrete mathematics (e.g., diagrams, graphs, matrices, propositional statements) are applied in the analysis, interpretation, communication, and solution of problems.

For example:

- representing finite data using a variety of techniques
- representing real-world situations and relationships using sequences and recurrence relations
- modeling and solving problems using graphs and matrices
- evaluating the use of computers and calculators to solve problems (e.g., developing and analyzing algorithms)

SUBAREA VI—ALGEBRA: CONSTRUCTED-RESPONSE ASSIGNMENT

The content to be addressed by the constructed-response assignment is described in Subarea II, Objectives 04–09.

MULTIPLE-CHOICE SECTION

This preparation guide provides sample multiple-choice questions and a sample written assignment for the test. The multiple-choice questions illustrate the objectives of the test—one sample question for each objective.

Three pieces of information are presented for each test question:

1. the number of the test objective that the sample question illustrates,
2. a sample test question,
3. an indication of the correct response and an explanation of why it is the best available response.

Keep in mind when reviewing the questions and response options that there is one best answer to each question. Remember, too, that each explanation offers one of perhaps many perspectives on why a given response is correct or incorrect in the context of the question; there may be other explanations as well.

On the following page are sample test directions similar to those that candidates see when they take the test.

SAMPLE TEST DIRECTIONS FOR MULTIPLE-CHOICE QUESTIONS

DIRECTIONS

This test booklet contains a multiple-choice section and a section with a single written assignment. You may complete the sections of the test in the order you choose.

Each question in the first section of this booklet is a multiple-choice question with four answer choices. Read each question CAREFULLY and choose the ONE best answer. Record your answer on the answer document in the space that corresponds to the question number. Completely fill in the space that has the same letter as the answer you have chosen. *Use only a No. 2 lead pencil.*

Sample Question: 1. What is the capital of New York?

A. Buffalo
B. New York City
C. Albany
D. Rochester

The correct answer to this question is C. You would indicate that on the answer document as follows:

1. Ⓐ Ⓑ ● Ⓓ

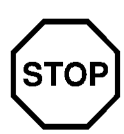
You should answer all questions. Even if you are unsure of an answer, it is better to guess than not to answer a question at all. You may use the margins of the test booklet for scratch paper, but you will be scored only on the responses on your answer document.

The directions for the written assignment appear later in this test booklet.

FOR TEST SECURITY REASONS, YOU MAY NOT TAKE NOTES OR REMOVE ANY OF THE TEST MATERIALS FROM THE ROOM.

The words "End of Test" indicate that you have completed the test. You may go back and review your answers, but be sure that you have answered all questions before raising your hand for dismissal. Your test materials must be returned to a test administrator when you finish the test.

If you have any questions, please ask them now before beginning the test.



DO NOT GO ON UNTIL YOU ARE TOLD TO DO SO.

Sample Mathematics Definitions and Formulas

Definitions and Formulas for Mathematics

LOGIC

$a \rightarrow b$	*a* implies *b*
$a \leftrightarrow b$	*a* if and only if *b*
$a \wedge b$	*a* and *b*
$a \vee b$	*a* or *b*
$\sim a$	not *a*
$A \cup B$	*A* union *B*
$A \cap B$	*A* intersect *B*
$\bar{A}$	complement of *A*
U	universal set
$\{\ \}$	empty set

ALGEBRA

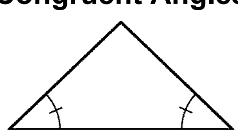
$i = \sqrt{-1}$	imaginary unit
$\bar{z}$	complex conjugate of *z*
A^{-1}	inverse of matrix *A*
$\vec{v}$	vector *v*

GEOMETRY

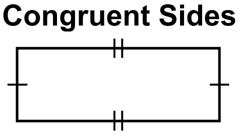
surface area of a sphere	$S = 4\pi r^2$
volume of a sphere	$V = \frac{4}{3}\pi r^3$

$\sim$	is similar to
$\cong$	is congruent to

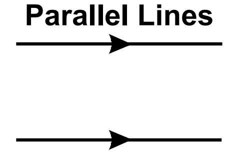
Congruent Angles



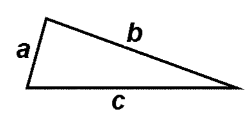
Congruent Sides



Parallel Lines



Heron's Formula

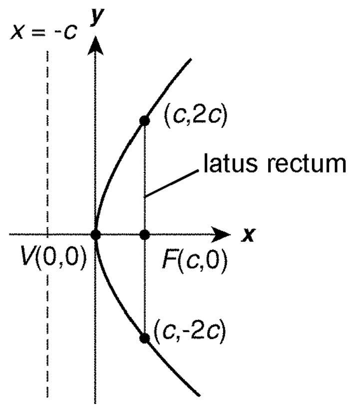


$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{where } s = \frac{a+b+c}{2}$$

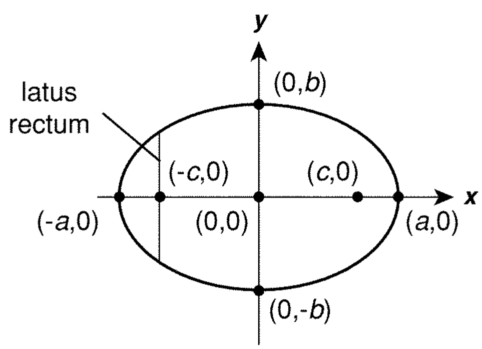
GEOMETRY (continued)

Parabola



$$(y-k)^2 = 4c(x-h)$$

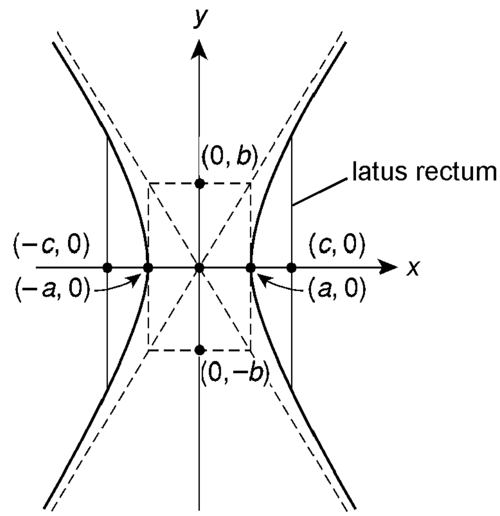
Ellipse



$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

where $c^2 = a^2 - b^2$

Hyperbola



$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

where $b^2 = c^2 - a^2$

Eccentricity of a Conic

$$e = \frac{c}{a}$$

Directrices of a Conic

$$x = \pm\frac{a}{e} = \pm\frac{a^2}{c}$$

TRIGONOMETRY

$\sin(\theta_1 \pm \theta_2) = \sin\theta_1 \cos\theta_2 \pm \cos\theta_1 \sin\theta_2$

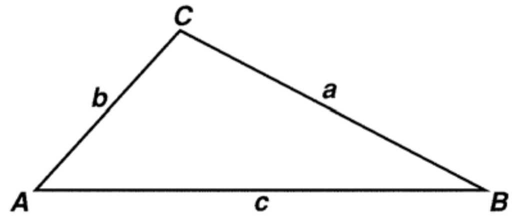
$\cos(\theta_1 \pm \theta_2) = \cos\theta_1 \cos\theta_2 \mp \sin\theta_1 \sin\theta_2$

$$\tan(\theta_1 \pm \theta_2) = \frac{\tan\theta_1 \pm \tan\theta_2}{1 \mp \tan\theta_1 \tan\theta_2}$$

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1 - \cos\theta}{2}}$$

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1 + \cos\theta}{2}}$$

$$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1 - \cos\theta}{1 + \cos\theta}}$$



Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Law of Cosines

$$c^2 = a^2 + b^2 - 2ab\cos C$$

STATISTICS

standard deviation of a sample mean $= \dfrac{\sigma}{\sqrt{N}}$

NOTES FOR MATHEMATICS TEST

In this examination, assume all functions are real valued functions unless otherwise noted.

In this examination, diagrams may not be drawn to scale.

SAMPLE MULTIPLE-CHOICE QUESTIONS, CORRECT RESPONSES, AND EXPLANATIONS

Objective 0001

Understand reasoning processes, including inductive and deductive logic and symbolic logic.

1. **Use the conjecture below to answer the question that follows.**

Conjecture: Every even natural number greater than 2 is equal to the sum of two prime numbers.

Which of the following would be a counterexample of the conjecture?

A. an even natural number greater than 2 that is not equal to the sum of two prime numbers

B. an even natural number greater than 2 that is equal to the sum of two prime numbers

C. an odd natural number greater than 2 that is equal to the sum of two prime numbers

D. an odd natural number greater than 2 that is not equal to the sum of two prime numbers

Correct Response: A. A counterexample of a conjecture is an example that satisfies the conditions of the conjecture (the hypothesis) but does not satisfy the conclusion. Therefore, a counterexample of the conjecture would be an example of an even natural number that is greater than 2 but is not equal to the sum of two prime numbers.

Objective 0002

Understand the meaning of mathematical concepts and symbols and how to communicate mathematical ideas in writing.

2. If $f(x) = \dfrac{3(x^2 - 2)}{x^2 + x - 6}$, then which of the following must be true?

A. $x \neq \sqrt{2}$

B. $x \neq \sqrt{3}$

C. $x \neq 2$

D. $x \neq 3$

Correct Response: C. A function of the form $\dfrac{P(x)}{Q(x)}$ must exclude from its domain values of x that result in $Q(x) = 0$, because when $Q(x) = 0$, the function $\dfrac{P(x)}{Q(x)}$ is undefined. Since $Q(x) = x^2 + x - 6 = (x + 3)(x - 2) = 0$ when $x = -3$ or 2, the numbers -3 and 2 must be excluded from the domain. Therefore, $x \neq 2$.

Objective 0003

Understand mathematical modeling and apply multiple mathematical representations to connect mathematical ideas and solve problems.

3. In order to calculate distances, the navigator of a small sailboat wants to make a list of square roots for the integers between 1 and 100, inclusive, by using the square root algorithm as few times as possible. Which of the following methods should the navigator use?

   A. Calculate the square roots of the integers between 1 and 10, inclusive, and use the property $\sqrt{a} + \sqrt{a} = 2\sqrt{a}$.

   B. Calculate the square roots of the prime numbers between 1 and 100, inclusive, and use the property $\sqrt{ab} = \sqrt{a}\sqrt{b}$.

   C. Calculate the square root of the odd integers between 1 and 100, inclusive, and use the property $\sqrt{(2n + 1)^2} = \sqrt{4n^2 + 4n + 1}$.

   D. Calculate the square root of the even integers between 1 and 50, inclusive, and use the property $\sqrt{2n^2} = n\sqrt{2}$.

Correct Response: B. The prime factorization theorem states that every integer can be uniquely factored into a product of prime numbers. Therefore, any integer between 1 and 100, inclusive, can be uniquely factored into a product of prime numbers, with each prime number less than 100. Application of the property $\sqrt{ab} = \sqrt{a}\sqrt{b}$ allows the computation of the square root of all the integers between 1 and 100, inclusive, without directly using the square root algorithm once the square roots of the prime factors are known.

Objective 0004

Understand principles and properties of the set of complex numbers and its subsets.

4. Which of the following is a geometric representation of an irrational number?

   A. the area of a rectangle with sides of length $\sqrt{12}$ and $\sqrt{3}$

   B. the coordinate of the point on the number line halfway between $\frac{3}{7}$ and $\frac{4}{7}$

   C. the length of the line segment connecting the points (0, 0) and (4, 5) in the coordinate plane

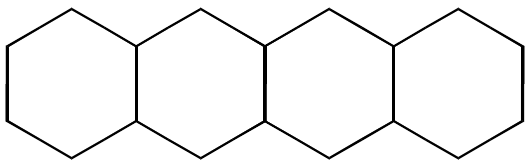
   D. the length of the diagonal of a rectangle with sides of length 3 and 4

Correct Response: C. An irrational number is a real number that cannot be written as $\frac{a}{b}$, where a and b are integers and $b \neq 0$. It follows from this definition that the decimal expansion of an irrational number does not terminate or repeat. The length of the line segment connecting the points (0, 0) and (4, 5) in the coordinate plane can be calculated by the distance formula. Applying the distance formula, $d = \sqrt{(4 - 0)^2 + (5 - 0)^2} = \sqrt{41} = 6.4031247\ldots$. Since $\sqrt{41}$ does not terminate or repeat, $\sqrt{41}$ is an irrational number.

Objective 0005

Understand the principles and properties of patterns and algebraic operations and relations.

5. **Use the figure below to answer the question that follows.**



Chains of regular polygons can be formed by placing individual polygons side by side. For example, the figure above shows a chain of four regular hexagons. Which of the following expressions represents the perimeter of a chain of *n* regular polygons if each polygon has *s* sides of length 1?

A. $(n + 1)(s)$

B. $(n + 2)(s - 2)$

C. $n(s - 2) + 2$

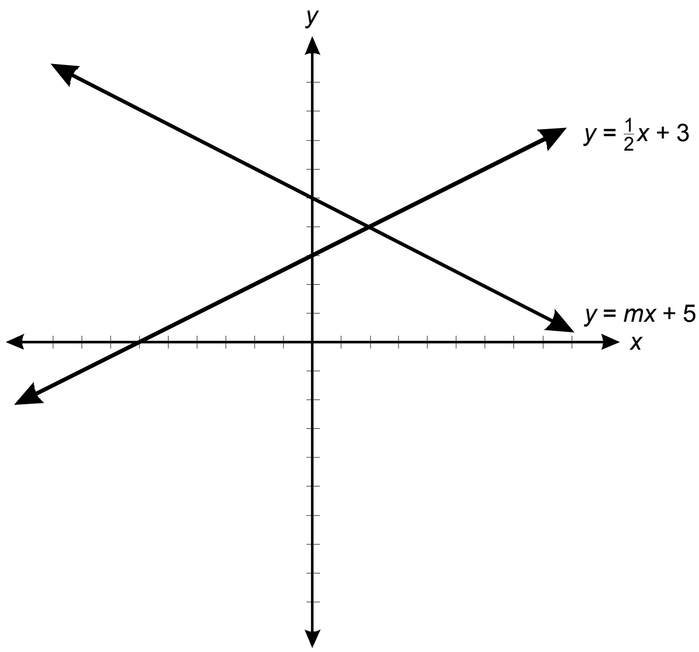
D. $n(\frac{s}{2} + 2)$

Correct Response: C. All of the polygons, except the two end polygons, have two shared sides that do not affect the perimeter. Each of the two end polygons has one shared side that does not affect the perimeter. The expression $n(s - 2)$ represents a perimeter that accounts for all of the polygons having two shared sides. To make up for the two unshared sides on the end polygons, 2 must be added to the expression. Therefore, $n(s - 2) + 2$ represents the perimeter.

Objective 0006

Understand the properties of linear functions and relations.

6. **Use the graph below to answer the question that follows**.



The above graph represents a system of linear equations. For what values of m will the solution to the system be in the first quadrant?

A. $m < \frac{1}{2}$

B. $m < 5$

C. $m > \frac{1}{2}$

D. $m > 5$

Correct Response: A. The graphic solution to a system of two linear equations is the point of intersection of the two lines. Since both lines are in the form $y = mx + b$, one line has a slope of $\frac{1}{2}$, and the slope of the second line, m, must be determined. In order for the two lines to intersect, the value of m cannot equal $\frac{1}{2}$. In order for the point of intersection to lie in the first quadrant, the value of m must be less than $\frac{1}{2}$.

Objective 0007

Understand the properties of quadratic and higher-order polynomial functions and relations.

7. A fireworks rocket is launched from a pit 10 feet below the earth's surface. The data in the table below represent the rocket's elevation above the ground t seconds after launch.

Time (sec.) Since Launch	**Elevation (ft.)**
0	–10
0.5	28
1.0	55
1.5	72
2.0	86
2.5	90
3.0	88
3.5	76
4.0	54
4.5	26

Which of the following equations best models the data?

A. $h(t) = -16t^2 + 80t - 10$

B. $h(t) = -16t^2 + 90t - 10$

C. $h(t) = -4.9t^2 + 70t - 10$

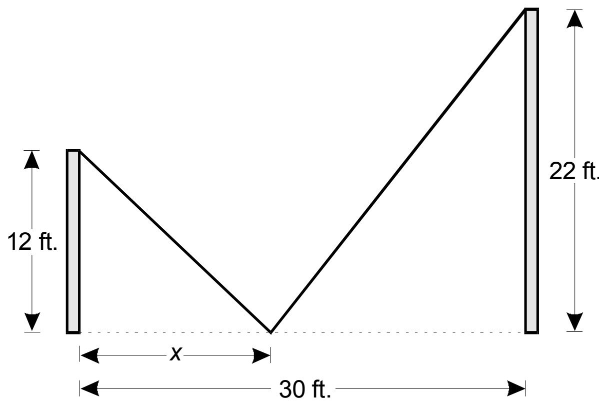
D. $h(t) = -12t^2 + 75t - 10$

Correct Response: A. Input the time and elevation data into two lists in a graphing calculator. Use the statistical features of the graphing calculator to find the quadratic regression model that best fits the data. The approximate model obtained is $h(t) = -15.98t^2 + 79.97t - 9.54$. Therefore, $h(t) = -16t^2 + 80t - 10$ best models the data.

Objective 0008

Understand the properties of rational, radical, and absolute value functions and relations.

8. **Use the diagram below to answer the question that follows.**



Two posts, one 12 feet high and the other 22 feet high, stand 30 feet apart. They are held in place by two wires, attached to a single stake, running from ground level to the top of each post. If x represents the distance from the base of the 12-foot post to the stake, which of the following expressions represents the total length of the wire in terms of x?

A. $\sqrt{x^2 + 144} + \sqrt{x^2 + 484}$

B. $\sqrt{x^2 + 144} + \sqrt{x^2 - 60x + 1384}$

C. $\sqrt{x^2 + 24x + 144} + \sqrt{x^2 + 44x + 484}$

D. $\sqrt{x^2 + 24x + 144} + \sqrt{x^2 - 60x + 1384}$

Correct Response: B. The Pythagorean theorem can be used to calculate the length of each portion of the wire. Let ℓ represent the length of the wire from the 12-foot post to the stake. Since $x^2 + (12)^2 = \ell^2$, then $\sqrt{x^2 + 144} = \ell$. Let r represent the length of the wire from the 22-foot post to the stake. The distance from the stake to the 22-foot post is $30 - x$. Since $(30 - x)^2 + (22)^2 = r^2$, then $\sqrt{(30 - x)^2 + 484} = \sqrt{x^2 - 60x + 1384} = r$. Therefore, the total length, $\ell + r$, of the wire is $\sqrt{x^2 + 144} + \sqrt{x^2 - 60x + 1384}$.

Objective 0009

Understand the properties of exponential and logarithmic functions.

9. A deposit of $250.00 is made in an interest-paying savings account. The table shows the amount of money in the account at the end of each of three equal payment periods. No money is deposited or withdrawn from the account, and the interest rate remains constant.

Deposit	$250.00
Period 1	$254.38
Period 2	$258.83
Period 3	$263.36

Which of the following expressions could be used to determine the amount of money in the account after n payment periods?

A. $250 + (0.0175)n$

B. $250 + 250(0.0175)n$

C. $250(0.0175)^n$

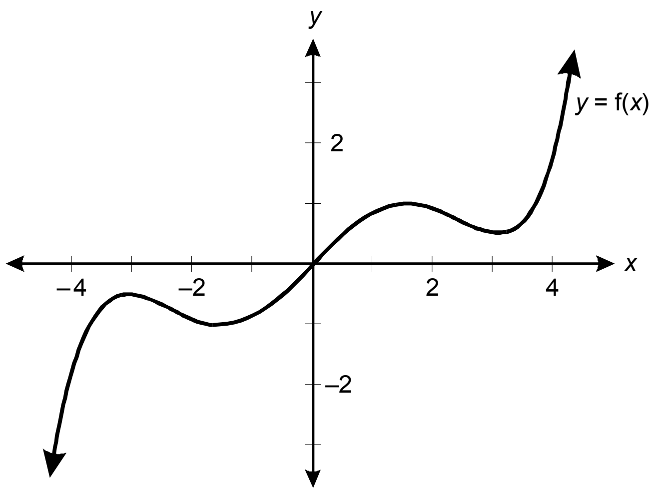
D. $250(1.0175)^n$

Correct Response: D. The account pays a constant interest rate at the end of each period. Let r = the interest rate. Then $250r = 4.38$ and $r = 0.0175$ rounded to the nearest ten-thousandth. The amount of money in the account at the end of Period 1 is $250 + 250(0.0175) = 250(1.0175)$. To facilitate the calculation, let $x = 250(1.0175)$. Then the amount of money at the end of Period 2 is $x + x(0.0175) = x(1.0175) = 250(1.0175)^2$. Continuing the pattern in this way shows that $250(1.0175)^n$ is the amount of money in the account after n payment periods.

Objective 0010

Understand principles, properties, and relationships involving trigonometric functions and their associated geometric representations.

10. **Use the graph below to answer the question that follows.**



The graph of the partial sum $f(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!}$ of the power series $\sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)!}$ is shown above. The graph of f(x) gives a good approximation of which of the following functions over the interval $-2 \le x \le 2$?

A. sin x

B. cos x

C. tan x

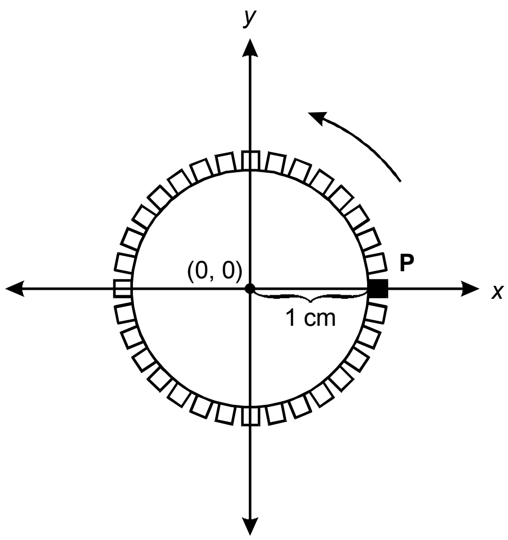
D. csc x

Correct Response: A. Of the choices given, the only trigonometric functions that have a root of 0 are sin x and tan x. The local maximum of 1 on the graph of f(x) occurs at approximately $x = 1.5$. The sine function has a maximum of 1 at $x = \frac{\pi}{2} \approx 1.57$, while the tangent function is undefined at $x = \frac{\pi}{2}$. Therefore, the graph of f(x) gives a good approximation of sin x over the interval $-2 \le x \le 2$. Note that the partial sum given by f(x) is the first three terms in the Taylor series of sin x centered at $x = 0$.

Objective 0011

Apply the principles and techniques of trigonometry to model and solve problems.

11. **Use the figure below to answer the question that follows.**



The gear above has a radius of 1 cm and rotates counterclockwise around the origin at a speed of 30 revolutions per second. At $t = 0$, point P has coordinates (1, 0). Which of the following functions could be used to describe the x-coordinate of P as a function of time, t, measured in seconds?

A. $f(t) = \sin(60\pi t)$

B. $f(t) = \sin(30\pi t)$

C. $f(t) = \cos(60\pi t)$

D. $f(t) = \cos(30\pi t)$

Correct Response: C. Since point P travels the circumference of a unit circle, the rectangular coordinates of point P are given by $(\cos\theta, \sin\theta)$, where θ is the angle made between the x-axis and the segment connecting P with the origin. Since one revolution = $360° = 2\pi$ radians, the point P revolves at a rate of 60π radians/second. Hence $\theta = 60\pi t$, since $\theta = 0$ at $t = 0$ and t is measured in seconds. Therefore, the x-coordinate of P as a function of time is $f(t) = \cos(60\pi t)$.

Objective 0012

Demonstrate an understanding of the fundamental concepts of calculus.

12. The number of people who become infected with a contagious disease at time t is represented by $y = N(t)$. Which of the following conditions indicates that the number of newly infected people is increasing, but at a slower rate than in the previous time period?

    A. $N'(t) > 0$ and $N''(t) > 0$

    B. $N'(t) < 0$ and $N''(t) > 0$

    C. $N'(t) > 0$ and $N''(t) < 0$

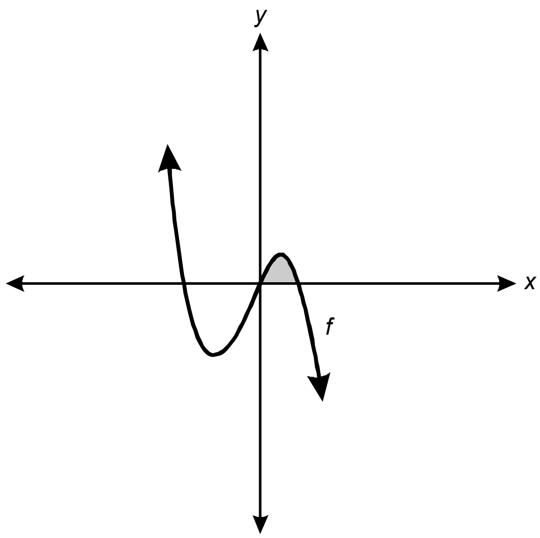
    D. $N'(t) < 0$ and $N''(t) < 0$

Correct Response: C. The first derivative of a function measures the rate of change of the function. Since the number of newly infected people is increasing, the first derivative, $N'(t)$, must be positive. The second derivative of a function measures the rate of change of the first derivative. Since the number of newly infected people is increasing at a slower rate, the first derivative is decreasing. Therefore, the second derivative, $N''(t)$, must be negative.

Objective 0013

Apply the principles and techniques of calculus to model and solve problems.

13. **Use the graph below to answer the question that follows.**



The graph above represents the function $f(x) = -x^3 - x^2 + 2x$.
What is the area of the shaded region?

A. $\frac{1}{6}$

B. $\frac{5}{12}$

C. $1\frac{5}{12}$

D. 3

Correct Response: B. The roots of $f(x)$ can be determined by factoring the function. Since $-x^3 - x^2 + 2x = -x(x + 2)(x - 1)$, the roots are 0, –2, and 1. The area of the shaded region is given by the definite integral $\int_0^1 (-x^3 - x^2 + 2x)dx$. Applying the rules of integration,

$$\int_0^1 (-x^3 - x^2 + 2x)dx = \left[-\frac{x^4}{4} - \frac{x^3}{3} + x^2\right]_0^1 = \left(-\frac{1}{4} - \frac{1}{3} + 1\right) - 0 = \frac{5}{12}.$$

Objective 0014

Understand and apply measurement principles.

14. A student estimates that human hair grows at a rate of 1.096×10^{-8} miles per hour. In making this estimate, the student assumed that human hair grows at a rate of:

    A. 0.5 inches per year.

    B. 5.6 inches per year.

    C. 6.1 inches per year.

    D. 9.6 inches per year.

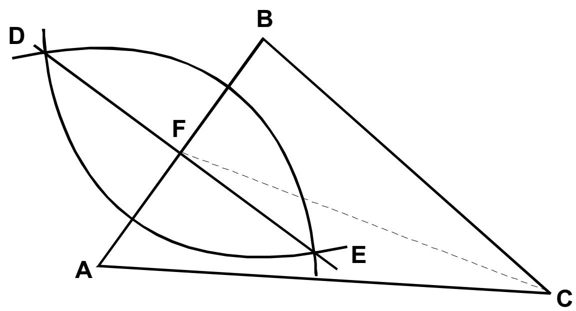
Correct Response: C. Use conversion factors to change miles per hour to inches per year:

$$\frac{1.096 \times 10^{-8}\text{ mi.}}{\text{hr.}} \times \frac{5280\text{ ft.}}{1\text{ mi.}} \times \frac{12\text{ in.}}{1\text{ ft.}} \times \frac{24\text{ hr.}}{1\text{ day}} \times \frac{365\text{ days}}{1\text{ year}} = 6.1\text{ inches per year.}$$

Objective 0015

Understand the principles and properties of axiomatic (synthetic) geometries.

15. **Use the figure below to answer the question that follows.**



Two arcs of the same radius are drawn, one centered at point *A* and the other at point *B*. Segment *DE* is then drawn, connecting the points where the arcs intersect, and intersecting segment *AB* at point *F*. Segment *FC* is then drawn from vertex *C* to point *F*. Which of the following best describes segment *FC*?

A. a perpendicular bisector

B. a median

C. an angle bisector

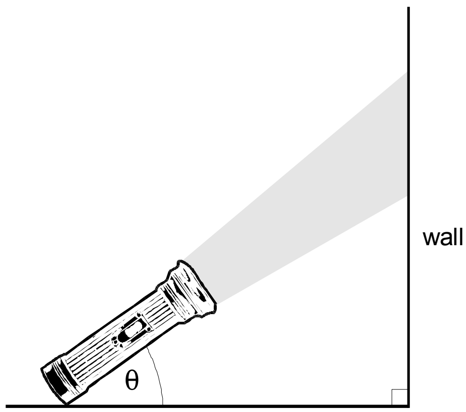
D. an altitude

Correct Response: B. The procedure described to produce segment *DE* is the compass-and-straightedge construction of the perpendicular bisector of line segment *AB*. Hence, *AF* = *BF* and *F* is the midpoint of segment *AB*. By definition, a median of a triangle is a segment from a vertex to the midpoint of the opposite side. Therefore, segment *FC* is a median.

Objective 0016

Understand the principles and properties of coordinate geometry.

16. **Use the diagram below to answer the question that follows.**



A flashlight has a circular face and is tilted at a non-zero acute angle, θ, to create a closed illuminated figure on a vertical wall as shown above. Which of the following equations could be used to represent the border of the figure if an *x*-*y* coordinate system is placed on the wall?

A. $\frac{x^2}{a^2} + \frac{y}{b} = 1$

B. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

C. $\frac{x^2}{a^2} + \frac{y^2}{a^2} = 1$

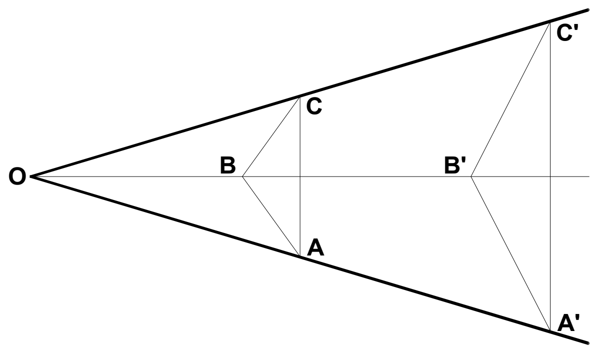
D. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Correct Response: B. The flashlight creates a cone of light, and the intersection of the wall (a plane) and the cone is a conic section. Since θ is an acute angle, the conic section is an ellipse. Placing a coordinate system on the wall with the ellipse centered at the origin results in $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, which is the equation of an ellipse in standard form.

Objective 0017

Apply mathematical principles and techniques to model and solve problems involving vector and transformational geometries.

17. **Use the diagram below to answer the question that follows.**



The transformation T of $\triangle ABC$ to $\triangle A'B'C'$ is a dilation with center at O. If the area of $\triangle A'B'C'$ is twice the area of $\triangle ABC$, what is the scale factor of T?

A. $\frac{1}{2}$

B. $\frac{\sqrt{2}}{2}$

C. $\sqrt{2}$

D. 2

Correct Response: C. A dilation is a similarity transformation. Therefore, $\triangle ABC \sim \triangle A'B'C'$. If the dilation has scale factor k, then the area of $\triangle A'B'C' = k^2 \times$ (the area of $\triangle ABC$) = $2 \times$ (the area of $\triangle ABC$). Therefore, $k^2 = 2$ and $k = \sqrt{2}$.

Objective 0018

Understand the principles, properties, and techniques related to sequence, series, summation, and counting strategies and their applications to problem solving.

18. A competitor in an event at a hot-air balloon festival throws a marker down from an altitude of 200 feet toward a target. The table below shows the relationship between the height of the marker above the ground and the time since the competitor threw the marker.

Time (sec.)	0	0.25	0.5	0.75	1.0	1.25
Height (ft.)	200	189	176	161	144	125

How many seconds after the competitor throws the marker will the marker hit the ground?

A. 2.0 seconds

B. 2.5 seconds

C. 3.0 seconds

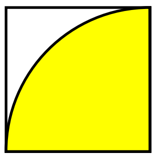
D. 3.5 seconds

Correct Response: B. In the first 0.25 seconds, the marker drops 11 feet. In the next 0.25 seconds, the marker drops 13 feet. And, in the next 0.25 seconds, the marker drops 15 feet. Continuing this pattern by extending the table, the marker will hit the ground (height = 0) after 2.5 seconds.

Objective 0019

Understand the principles, properties, and techniques of probability and their applications.

19. **Use the diagram below to answer the question that follows.**



A target consists of a square region in which a quarter circle is drawn and shaded. The radius of the circle is equal to one side of the square. A computer program calculates pi (π) by simulating darts being fired randomly at the target. A success is defined as a dart falling within the shaded area. If the computer obtains a value of 3.12 after 1000 shots at the target, how many darts landed in the shaded area? Assume that the probability of hitting the target is one.

A. 312

B. 624

C. 750

D. 780

Correct Response: D. Let N represent the number of darts that land in the shaded area. The relative frequency of the landing darts is $f = \frac{N}{1000}$. The probability that a randomly fired dart lands in the quarter circle is equal to the ratio of the area of the quarter circle to the area of the square. Since the number of shots is relatively large, this probability is approximately equal to the relative frequency. Let s represent the length of the side of the square. Since the area of a circle is πs^2, we obtain $\frac{N}{1000} = \frac{(\pi/4)s^2}{s^2}$. Substituting 3.12 for the value of the value obtained for π by the computer, simplifying and solving for N results in $N = 780$.

Objective 0020

Understand the principles, properties, and techniques of data analysis and statistics.

20. A student received a grade of 64% on a test weighted as 25% of the final grade, a grade of 80% on a test weighted as 35% of the final grade, and a grade of 70% on a test weighted as 20% of the final grade. The student has not yet taken the remaining test weighted as 20% of the final grade. What is the student's current percentage grade?

    A. 71.5%

    B. 72.5%

    C. 73.5%

    D. 74.5%

Correct Response: B. The student has earned 25(0.64) + 35(0.80) + 20(0.70) = 58 of the 80 percentage points at this time. Since $\frac{58}{80} = 0.725$, the student's current percentage grade average is 72.5.

Objective 0021

Understand how techniques of discrete mathematics (e.g., diagrams, graphs, matrices, propositional statements) are applied in the analysis, interpretation, communication, and solution of problems.

21. **Use the information below to answer the question that follows.**

A menu lists three kinds of soup, five main courses, and two desserts. If a meal consists of a soup, a main course, and a dessert, how many different meals are possible?

Which of the following would be most useful in solving the problem above?

A. a frequency diagram

B. conditional probability

C. prime factorization

D. a tree diagram

Correct Response: D. A tree diagram can be used to solve counting problems. A tree diagram consists of branches leaving a starting point and possible additional branches leaving the endpoints of other branches. Each path from a starting point to an endpoint represents a possible choice. In this case, three branches could represent the soups, five branches from each soup endpoint could represent the main courses, and two branches from each main course endpoint could represent the desserts, for a total of thirty branches in the tree.

WRITTEN ASSIGNMENT SECTION

On the following pages are:

- Sample test directions for the written assignment section
- A sample written assignment
- An example of a strong response to the assignment
- The performance characteristics and scoring scale
- An evaluation of the strong response

On the actual test, candidates will be given a different written assignment from the one provided as a sample in this preparation guide.

SAMPLE TEST DIRECTIONS FOR THE WRITTEN ASSIGNMENT

DIRECTIONS FOR THE WRITTEN ASSIGNMENT

This section of the test consists of a written assignment. You are to prepare a written response of about one to two pages on the assigned topic. ***The assignment can be found on the next page***. You should use your time to plan, write, review, and edit your response to the assignment.

Read the assignment carefully before you begin to write. Think about how you will organize your response. You may use any blank space provided on the following pages to make notes, write an outline, or otherwise prepare your response. ***However, your score will be based solely on the response you write on the lined pages of your answer document.***

Your response will be evaluated on the basis of the following criteria.

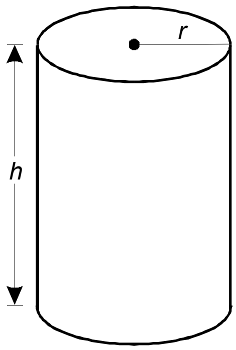
- **PURPOSE:** Fulfill the charge of the assignment.
- **APPLICATION OF CONTENT:** Accurately and effectively apply the relevant knowledge and skills.
- **SUPPORT:** Support the response with appropriate examples and/or sound reasoning reflecting an understanding of the relevant knowledge and skills.

Your response will be evaluated on the criteria above, not on writing ability. However, your response must be communicated clearly enough to permit valid judgment of your knowledge and skills. The final version of your response should conform to the conventions of edited American English. This should be your original work, written in your own words, and not copied or paraphrased from some other work.

Be sure to write about the assigned topic. Please write legibly. You may not use any reference materials during the test. Remember to review what you have written and make any changes that you think will improve your response.

WRITTEN ASSIGNMENT

Use the diagram and the information below to complete the exercise that follows.



The diagram shows a right cylinder with a radius *r* and height *h*. A beverage company wants to make a right cylindrical can that holds 500 cm^3 of juice. Assume the thickness of the material used to make the can is negligible.

- Use your knowledge of volume and surface area to derive a function A(*r*) that represents the surface area of the can in terms of the radius, *r,* of its base;

- use your calculator to produce a graph of A(*r*) that shows the intercepts, extrema, and asymptotic behavior of the function over the set of real numbers, sketch the graph you obtained, and identify the window you used;

- state any restriction on the domain of the function so that it represents the physical model of the can; and

- use your calculator to find the dimensions of the can to the nearest tenth of a centimeter that will minimize the quantity of material needed to manufacture the can.

Be sure to show your work and explain the steps you used to obtain your answers.

STRONG RESPONSE TO THE SAMPLE WRITTEN ASSIGNMENT

- A beverage company wants to make a right cylindrical can that holds 500 cm^3 of juice. Derive $A(r)$ that represents the surface area of the can. The surface area, $A(r)$, is equal to: $2\pi rh + 2\pi r^2$.

  Since we have a volume of 500 cm^3,

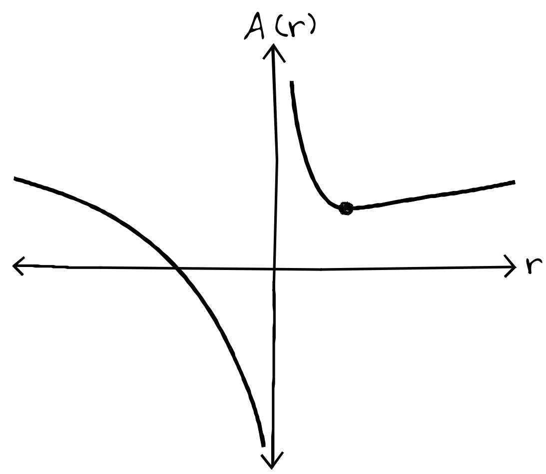
$$V = \pi r^2 h$$

$$500 = \pi r^2 h$$

$$h = \frac{500}{\pi r^2} \quad \Rightarrow \text{ substitute into } A(r)$$

$$A(r) = 2\pi r\left(\frac{500}{\pi r^2}\right) + 2\pi r^2 = \frac{1000}{r} + 2\pi r^2$$

- A graph of this function looks like:



  The window used was of the form: $r_{min} = -10$ $A_{min} = -1000$
  $r_{max} = 10$ $A_{max} = 1000$
  $r_{scl} = 1$ $A_{scl} = 100$

  The vertical asymptote is $r = 0$.

  The graph has an r-intercept of -5.41.

  The local minimum is at (4.3, 348.7).

- The domain of this function is restricted to $(0, \infty)$ because we need a positive radius for the can.

- To minimize surface area, set the first derivative equal to zero.

$$A'(r) = \frac{-1000}{r^2} + 4\pi r = 0$$

$$-1000 + 4\pi r^3 = 0$$

$$r^3 = \frac{1000}{4\pi} = 79.578 \Rightarrow r = 4.301 \text{ cm}$$

$$A''(r) = \frac{2000}{r^3} + 4\pi \Rightarrow A''(4.301) = \frac{2000}{(4.301)^3} + 4\pi > 0 \Rightarrow$$ $A(r)$ has a local minimum at $r = 4.301$.

When $r = 4.301$, $h = \dfrac{500}{\pi(4.301)^2} = 8.6036$

So, a 500 cm^3 can that requires the least amount of material to manufacture has a radius of 4.3 cm and a height of 8.6 cm.

PERFORMANCE CHARACTERISTICS AND SCORING SCALE

Performance Characteristics

The following characteristics guide the scoring of responses to the written assignment.

Purpose:	Fulfill the charge of the assignment.
Application of Content:	Accurately and effectively apply the relevant knowledge and skills.
Support:	Support the response with appropriate examples and/or sound reasoning reflecting an understanding of the relevant knowledge and skills.

Scoring Scale

Scores will be assigned to each response to the written assignment according to the following scoring scale.

Score Point	Score Point Description
4	**The "4" response reflects a thorough command of the relevant knowledge and skills.** • The response completely fulfills the purpose of the assignment by responding fully to the given task. • The response demonstrates an accurate and highly effective application of the relevant knowledge and skills. • The response provides strong support with high-quality, relevant examples and/or sound reasoning.
3	**The "3" response reflects a general command of the relevant knowledge and skills.** • The response generally fulfills the purpose of the assignment by responding to the given task. • The response demonstrates a generally accurate and effective application of the relevant knowledge and skills. • The response provides support with some relevant examples and/or generally sound reasoning.
2	**The "2" response reflects a partial command of the relevant knowledge and skills.** • The response partially fulfills the purpose of the assignment by responding in a limited way to the given task. • The response demonstrates a limited, partially accurate and partially effective application of the relevant knowledge and skills. • The response provides limited support with few examples and/or some flawed reasoning.
1	**The "1" response reflects little or no command of the relevant knowledge and skills.** • The response fails to fulfill the purpose of the assignment. • The response demonstrates a largely inaccurate and/or ineffective application of the relevant knowledge and skills. • The response provides little or no support with few, if any, examples and/or seriously flawed reasoning.

EVALUATION OF THE STRONG RESPONSE

This response is considered a strong response because it reflects a thorough command of relevant knowledge and skills.

Purpose. The surface area of the can, A(r), is correctly derived in terms of r. The graph of A(r) is sketched over the set of real numbers and the window used to generate the graph is given. The vertical asymptote, r-intercept, and local minimum are clearly stated and displayed on the graph. The domain of the function, A(r), is clearly stated. The candidate accurately determines, to the nearest tenth of a centimeter, the dimensions of the can that minimize the amount of material needed for its construction.

Application of Content. The formula for the volume of the can is appropriately set equal to 500 and accurately solved for h. The function for h, derived from volume, is appropriately substituted into A(r), and A(r) accurately simplified. The graph of A(r) is sketched appropriately and all relevant information about the graph stated accurately. The domain of A(r) is understood to include only positive values of r. The candidate uses calculus to determine the minimum value of A(r) by correctly finding the first derivative of A(r), equating it to zero, and correctly solving for r. The value of r is accurately calculated using the π key on a calculator, rather than an approximation such as 3.14. The second derivative is correctly determined and evaluated at $r = 4.301$ to ensure that a minimum exists at that value. The corresponding value of h is correctly evaluated for $r = 4.301$.

Although calculus was used to determine the dimensions that minimize the surface area of the can, the correct value of r could have been given as the r-coordinate of the local minimum of the graph of A(r) (from the answer in the second bullet). This value is found using the TRACE command on a graphing calculator.

Support. The steps involved in determining h in terms of r and in determining A(r) are clearly shown. The reason that only positive values are included in the domain of A(r) is clearly stated. The candidate explains why the first derivative is found and set equal to zero. The way in which $A'(r) = 0$ is solved is clearly presented. The reason that $r = 4.301$ results in a minimum surface area is appropriately given as a result of the second derivative test. The determination of the value of h, for $r = 4.301$, is clearly presented.

If the TRACE command is used to find r, appropriate reasoning would be provided as to why this value results in a minimum surface area.