Math 6250 Quiz 4

Name:____

1. Show the following are monotone or not. State whether they are monotone increasing, monotone decreasing or not monotone. And prove it.
   (a) $a_n = \frac{1}{n}$.
   (b) Defined recursively as $a_1 = 1$ and $a_{n+1} = 1 + \frac{1}{a_n}$.
   (c) Defined recursively as $a_1 = 1$ and $a_{n+1} = 1 + \frac{a_n}{a_n+1}$.
   (d) $a_n = \frac{-1}{n}$.
   (e) What do the sequences from 1b and 1c have to do with a well known sequence?

2. Prove the Monotone convergence Theorem: That is
   If (a_n) is a bounded and monotone sequence then (a_n) converges.

3. Use the Monotone Convergence Theorem to show that: the sequence defined as $a_1 = 1$ and $a_{n+1} = 1 + \frac{a_n}{a_n+1}$ converges.

4. Prove with $\varepsilon - N$ proof that $a_n = \frac{2n+1}{3n+5}$ converges.

5. Prove with $\varepsilon - N$ proof that the sequence defined below is not Cauchy.

$$a_n = \sum_{j=1}^{n} \frac{1}{j}$$

   So $a_1 = \sum_{j=1}^{1} \frac{1}{j} = \frac{1}{1} = 1$, $a_2 = \sum_{j=1}^{2} \frac{1}{j} = \frac{1}{1} + \frac{1}{2} = \frac{3}{2}$ and $a_3 = \sum_{j=1}^{3} \frac{1}{j} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} = \frac{11}{6}$.

6. Let (a_j) be a sequence in $\mathbb{C}$ so that for all pairs of integers $0 < M < N$ we have

$$|a_M - a_{M+1}| + |a_{M+1} - a_{M+2}| + |a_{M+2} - a_{M+3}| + \cdots + |a_{N-1} - a_N| \leq 1.$$

   Then (a_j) converges.