Math 2320 - Notes on Power Series

1 Power Series

1. Compute the Interval of Convergence for the given power series.
   (a) $\sum_{n=0}^{\infty} \frac{1}{n} x^n$
   (b) $\sum_{n=0}^{\infty} \frac{1}{2^n} x^n$
   (c) $\sum_{n=0}^{\infty} \frac{1}{n} (x+1)^n$
   (d) $\sum_{n=0}^{\infty} \frac{n}{2^n} (x+1)^n$
   (e) $\sum_{n=0}^{\infty} \frac{1}{n^2 3^n} x^n$

2 Taylor Series

2. Compute the Taylor Series for $f(x)$ at the point $x = a$ from the formula.
   - $f(x) = \sin(x)$ and $a = 0$
   - $f(x) = \sin(x)$ and $a = \pi/2$
   - $f(x) = e^{2x}$ and $a = 0$
   - $f(x) = x^3 - 1$ and $a = 1$
   - $f(x) = \ln(1 - x)$ and $a = 0$
3. Compute the Taylor Series for $f(x)$ at the point $x = 0$ from one of the known series.
   - $f(x) = \sin(x^2)$
   - $f(x) = \frac{\sin(x) - x}{x^3}$
   - $f(x) = e^{2x}$
   - $f(x) = \frac{1}{1-x}$
   - $f(x) = e^{ix}$ where $i^2 = -1$
   - $f(x) = \cos(x) + i \sin(x)$ where $i^2 = -1$

3 Some Formulas

Taylor's Formula

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f^{(3)}(a)}{3!}(x-a)^3 + \cdots$$

Some Known Series

- $\sin(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!}x^{2k+1} = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \cdots$
- $\cos(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!}x^{2k} = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \cdots$
- $e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$
- $\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k = 1 + x + x^2 + x^3 + x^4 + x^5 + \cdots$