MA 2320: Series Tests

- **Telescoping**
- **Geometric** - Given $\sum_{k=0}^{\infty} r^k$

if $\|r\| < 1$	then the series converges to $\frac{1}{1-r}$
if $\|r\| \geq 1$	then the series diverges.

- **Divergence Test** - Given $\sum a_k$.

if $\lim a_k \neq 0$	then the series diverges.

- **Integral Test** - Given $\sum_1^{\infty} a_k$ where $a_k = f(k)$ and $f(x)$ is continuous and decreasing.

if $\int_1^{\infty} f(x)dx$ converges	then $\sum_1^{\infty} a_k$ converges.
if $\int_1^{\infty} f(x)dx$ diverges	then $\sum_1^{\infty} a_k$ diverges.

- **P-SeriesTest** - Given $\sum_1^{\infty} \frac{1}{k^p}$.

if $p > 1$	then $\sum_1^{\infty} a_k$ converges.
if $p \leq 1$	then $\sum_1^{\infty} a_k$ diverges.

- **Ratio Test** - Given $\sum a_k$. Compute $r = \lim \frac{a_{k+1}}{a_k}$

if $r < 1$	then $\sum_1^{\infty} a_k$ converges.
if $r > 1$	then $\sum_1^{\infty} a_k$ diverges.
if $r = 1$	then the ratio test is inconclusive.

- **Root Test** - Given $\sum a_k$. Compute $r = \lim \sqrt[k]{a_k}$

if $r < 1$	then $\sum_1^{\infty} a_k$ converges.
if $r > 1$	then $\sum_1^{\infty} a_k$ diverges.
if $r = 1$	then the root test is inconclusive.

- **Comparison Test** - Given $\sum a_k$. We use a known series, say $\sum b_k$.

if $0 \leq a_k \leq b_k$ and $\sum b_k$ converges	then $\sum_1^{\infty} a_k$ converges.
if $0 \leq b_k \leq a_k$ and $\sum b_k$ diverges	then $\sum_1^{\infty} a_k$ diverges.

- **Limit Comparison Test** - Given $\sum a_k$. We use a known series, say $\sum b_k$. Then compute $L = \lim \frac{a_k}{b_k}$.

if $0 < L < \infty$	then $\sum a_k$ and $\sum b_k$ do the same thing. That is, they both converge or they both diverge.
if $L = 0$ or $L = \infty$	then the test is inconclusive.

- **Alternating Series Test** - Given $\sum (-1)^k a_k$ where a_k is decreasing and positive.

if $\lim a_k = 0$	then the series converges.