CHEBYCHEV

(1) Let X be a R.V. s.t. $\mu_X = 7$ and $\sigma_X = 2$

(a) Find a bound on $Pr(4 \leq X \leq 10)$

(b) Find a bound on $Pr(X > 11)$

(2) Let X be a R.V. s.t. $\mu_X = 7$, $\sigma_X = 2$ and X has a symmetric distr.

(a) Find bound on $Pr(4 \leq X \leq 10)$

(b) Find bou on $Pr(X > 11)$

(3) Let X be Poisson with $\lambda = 7$

(a) Using Chebychev find a bound on

$Pr(3 \leq X \leq 11)$

(b) Using Poisson compute

$Pr(3 \leq X \leq 11)$.

(4) Let X be R.V with pdf $f(x) = \frac{1}{7} e^{-x/7}$

(a) Find $E(X)$ and $E(X^2)$

(b) Find σ

(c) Compute $Pr(3 \leq X \leq 11)$ direct

(d) use Chebychev to get a bound on

$Pr(3 \leq X \leq 11)$

CLT

(5) Let $X_1, X_2, \ldots X_n$ be iid with $\mu_x = 0.03$

AND $\sigma_x = 0.0291$

(a) What is $\mu_{\bar{X}}$ AND $\sigma_{\bar{X}}$

where $\bar{X} = \frac{X_1 + \ldots + X_n}{n}$?

(b) If $n = 2500$, use CLT to estimate

$$P_r\left(\bar{X} > \frac{80}{2500}\right)$$

(6) Let $X_1, X_2, \ldots, X_n$ be iid with μ_X unknown

and $\sigma_X = .02$

use CLT to find n s.t

$$P_r\left(|\bar{X}_n - \mu| \leq .1\right) = .95$$

ANSWERS

① (a) $Pr(4 \le X \le 10) = Pr(4-7 \le X-7 \le 10-7)$

$= P(-3 \le X-7 \le 3) = P(|X-7| \le 3) \underset{\text{C.T.}}{\geq} 1 - \frac{1}{k^2} = 1 - \frac{1}{(\frac{3}{2})^2}$

$k\sigma = 3$

$k(2) = 3$

$k = \frac{3}{2}$

(b) $Pr(X > 11)$

by C.T.

$Pr(\underbrace{|X-7| < 4}_{-11 < X < 11}) \geq 1 - \frac{1}{2^2} = \frac{3}{4}$

$k\sigma = 4$

$k(2) = 4$

$k = 2$

$\Rightarrow P(X > 11 \text{ or } X < -11) < \frac{1}{4}$

BUT WE CAN ONLY SAY

$P(X > 11) < \frac{1}{4}$.

② (a) Same as 1(a)

② (b)

$$Pr(\underbrace{|X-7| \le 4}) \ge 1 - \frac{1}{k^2} = \frac{3}{4}$$

$-11 < x < +11$

$k\sigma = 4$

$k(2) = 4$

$k = 2$

But since X has a symmetric dist. we know

$P(x < -11) = P(x > 11)$

So $P(x < -11) + P(x > 11) < \frac{1}{4}$

$\Rightarrow \boxed{P(x > 11) < \frac{1}{8}}$

③ Poisson $\lambda = 7 \Rightarrow \mu = 7$

$\& \ \sigma^2 = 7$

a) $P(3 \le X \le 11) = P(-4 \le X - 7 \le 4) \ge 1 - \frac{1}{\left(\frac{4}{\sqrt{7}}\right)^2}$

$k\sigma = 4$

$k\sqrt{7} = 4$

$k = \frac{4}{\sqrt{7}}$

$= 1 - \frac{7}{16} = \boxed{\frac{9}{16}}$

(b) $P(3 \le x \le 11) = e^{-7}\left[\frac{7^3}{3!} + \cdots + \frac{7^{11}}{11!}\right] \approx 0.917$

(4.) (a) $E(x) = \int x f(x)\,dx = \int_0^\infty x \cdot \frac{1}{7} e^{-x/7} dx$

$= \quad = 7$

$E(x^2) = \int x^2 f(x)\,dx = \quad = 98$

(b) so $VAR(x) = E(x^2) - E(x)^2 = 98 - (7)^2 = 49$

$\sigma = \sqrt{VAR(x)} = \sqrt{49} = 7$

(c) $P_r(3 \le x \le 11) = \int_3^{11} f(x)\,dx = \int_3^{11} \frac{1}{7} e^{-x/7} dx$

$= -e^{-x/7} \Big|_3^{11} = -e^{-11/7} + e^{-3/7}$

(d) $P_r(\underbrace{3 \le x \le 11})$

$|x-7| < 4$

$k\sigma = 4$

$k7 = 4$

$k = \frac{4}{7}$

$k \le 1$

CHEBYSHEV SAYS NOTHING

!

$1 - \frac{1}{k^2} = 1 - \frac{1}{(\frac{4}{7})^2} < 0$

(5.) (a) $\mu_{\bar{x}} = \mu_x = 0.03$

$$\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}} = \frac{0.0291}{\sqrt{n}}$$

(b) Let $n = 2500$ then

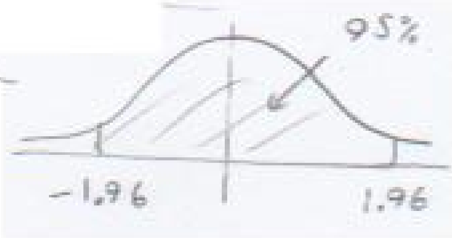
$$P_r\left(\bar{x} > \frac{80}{2500}\right)$$

$$Z = \frac{\bar{X} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{\frac{80}{2500} - 0.03}{\frac{0.0291}{\sqrt{2500}}} = 3.43$$

$$P(Z > 3.43) = .0003$$

(6.) $\sigma_{\bar{x}} = .02$

$$\underbrace{P(|\bar{X}_n - \mu| \le \boxed{.1})}= .95$$



$$Z = \frac{|X - \mu|}{\sigma_{\bar{X}_n}} = \frac{.1}{\sigma_{\bar{X}_n}} = 1.96$$

$$\frac{.1}{\frac{.2}{\sqrt{n}}} = 1.96$$

$$\frac{\sqrt{n}}{.2}(.1) = 1.96$$

$$\sqrt{n} = (1.96) * 2$$

$$\boxed{n \approx 16}$$