10. Let the life span of a light bulb be given by the pdf $f(x) = ce^{-x/7}$ where $x > 0$.

    Solution:

    (a) Is this random variable discrete or continuous? why?
    Continuous, since the values $x > 0$ is an interval.

    (b) Compute c

$$\begin{aligned}\int_0^\infty f(x)\,dx &= \int_0^\infty ce^{-x/7}\,dx \\ &= -7ce^{-x/7}|_0^\infty = -7ce^{-\infty/7} - -7ce^{-0/7} \\ &= 0 + 7c\end{aligned}$$

    Set equal to 1 to get.

$$c = 1/7$$

    So $f(x) = \frac{1}{7}e^{-x/7}$

    (c) Compute $P(X < 2)$ and $P(X \leq 2)$. So

$$\begin{aligned}P(X < 2) = \int_0^2 f(x)\,dx = \int_0^\infty \frac{1}{7}e^{-x/7}\,dx \\ &= -e^{-x/7}|_0^2 = -e^{-2/7} - -e^{-0/7} \\ &= -e^{-2/7} + 1\end{aligned}$$

    So $P(X < 2) = -e^{-2/7} + 1$ and $P(X \leq 2) = -e^{-2/7} + 1$.

    (d) Compute $P(X < 2|x \leq 2)$.

$$P(X < 2|X \leq 2) = \frac{X < 2 \cap X \leq 2}{X \leq 2} = \frac{X \leq 2}{X \leq 2} = 1.$$

    This particular question is more interesting for number 9.

1 Some Proofs

11. Prove: If $P(A) > P(A|B)$ then $P(B) > P(B|A)$. **Solution:** We did this in class

12. Prove: If A is independent of B then $P(A \cap B) = P(A)P(B)$. **Solution:** Assume A is independent of B, thus

$$P(A|B) = P(A). \tag{1}$$

    Also by definition we have

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \tag{2}$$

Putting equations (1) and (2) we have

$$P(A) = \frac{P(A \cap B)}{P(B)} \tag{3}$$

So $P(A \cap B) = P(A)P(B)$.