MA 5230 Test 1

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Show all work and justify your answers.

1. Use the list of numbers below

$$-1, 0, 0, 1, 5$$

   (a) What is the five number summary? What is the IQR?

   (b) Are there any outliers?

   (c) Compute $\bar{x}$ and s for the list.

(a) MIN $= -1$, $Q_1 = \frac{0+-1}{2} = -\frac{1}{2}$, median $= 0$, $Q_3 = \frac{1+5}{2} = 3$, Max $= 5$

$IQR = Q_3 - Q_1 = 3 - -\frac{1}{2} = 3.5$

(b) $Q_1 - 1.5 * IQR = -\frac{1}{2} - (1.5)(3.5) = -.5 - 3.5 - 1.75 = -5.75$

$Q_3 + 1.5 * IQR = 3 + (1.5)(3.5) = 3 + 3.5 + 1.75 = 8.25$

No outliers

(c) $\bar{X} = \frac{-1+0+0+1+5}{5} = 1$

$s^2 = \frac{1}{5-1}\left[(1--1) + 1^2 + 1^2 + 0^2 + 4^2\right] = \frac{1}{4}\{4+1+1+16\}$

$= \frac{22}{4} = \frac{11}{2}$

$s = \sqrt{\frac{11}{2}}$

2. I have a pair of fair dice. We roll the two dice and add the numbers on the faces.

   (a) What is the probability that the sum is a 6?

   (b) What is the probability that the sum is a 6 given that that the first roll is a 4?

   (c) Is the sum is a 6 independent of the first roll is a 4?

   (d) Is the sum is a 5 independent of the first roll is a 4?

(a) $P(S=6) = P((1\,5)\ (2\,4)\ (3\,3)\ (4\,2)\ (5\,1)) = \frac{5}{36}$

(b) $P(S=6 \mid F=4) = \dfrac{P((4,2))}{P(4,1\ \ 42\ \ 43 \cdots\ \ 46)} = \dfrac{\frac{1}{36}}{\frac{6}{36}} = \frac{1}{6}$

(c) $P(F=4) = \frac{6}{36} = \boxed{\frac{1}{6}}$

$P(F=4 \cap S=6)$

$P(F=4 \mid S=6) = \dfrac{P(4,2)}{P(S=6)} = \dfrac{\frac{1}{36}}{\frac{5}{36}} = \boxed{\frac{1}{5}}$

$P(F=4) \neq P(F=4 \mid S=6) \Rightarrow$ $F=4$ is NOT INDEPENDENT of $S=6$

(d) $P(S=5) = \frac{4}{36} \leftarrow \{(1,4)\ (2,3),\ (3,2)\ (4,1)\}$

$P(S=5 \mid F=4) = \dfrac{P(S=5 \cap F=4)}{P(F=4)} = \dfrac{\frac{1}{36}}{\frac{1}{6}} = \frac{1}{6}$

$P(S=5) \neq P(S=5 \mid F=4)$

So $F=4 \not\perp S=5$

is NOT INDEPENDENT

3. We will examine a test that detects a certain type of cancer.

   (a) If you have cancer the test is 99% accurate
   (b) If you are cancer free the test is 98% accurate

   Assume .01 % of the population has this cancer. Assume we test everyone
   Find.

   (a) probability that someone who is cancer free tests positive.
   (b) probability that someone who tests positive is cancer free.

(a) $\underset{\substack{\uparrow \\ \text{CANCER FREE}}}{P(C^c)} = 99.99\%$

$\boxed{\underset{\substack{\uparrow \\ \text{positive test}}}{P(P} \mid C^c) = 2\%}$

(b) $$P(C^c \mid P) = \frac{P(P \mid C^c) \cdot P(C^c)}{P(P \mid C^c) \cdot P(C^c) + P(P \mid C) \cdot P(C)}$$

$$= \frac{(0.02)(.9999)}{(0.02)(.9999) + (.99)(.0001)}$$

$$= 99.5\%$$

4. An urn has three red balls, five green balls and seven purple balls.

   (a) If we draw three balls from the urn (without replacement), what is the probability of drawing three green balls?

   (b) If we draw three balls from the urn (without replacement), what is the probability of drawing exactly two green balls?

(a) $P(G=3) = \dfrac{{}_5C_3 \quad {}_{15}C_0}{{}_{15}C_3} = \dfrac{\frac{5!}{3!\,2!}(1)}{\frac{20!}{17!\,3!}}$

(b) $P(G=2) = \dfrac{{}_5C_2 \quad {}_{15}C_1}{{}_{15}C_3} = \dfrac{\frac{5!}{3!\,2!} \cdot \frac{15!}{14!\,1!}}{\frac{20!}{17!\cdot 3!}}$

5. Let the pdf be defined as $f(n) = c(n+2)$ for where $n \in \{0, 2, 4\}$.

(a) Compute c

(b) Compute $P(N \geq 2)$.

(c) Compute $P(N = 2 | N \geq 2)$.

(a) $f(0) + f(2) + f(4) = 1$

$c(0+2) + c(2+2) + c(4+2) = 1 \Rightarrow \boxed{c = \frac{1}{12}}$

THUS $f(n) = \frac{1}{12}(n+2)$

(b) $P(N \geq 2) = f(2) + f(4)$

$= \frac{1}{12}(2+2) + \frac{1}{12}(2+4) = \boxed{\frac{10}{12}}$

(c) $P(N=2 | N \geq 2) = \dfrac{P(N=2 \cap N \geq 2)}{P(N \geq 2)} = \dfrac{P(N=2)}{P(N \geq 2)} = \dfrac{\frac{1}{12}(4)}{\frac{1}{12}(10)} = \frac{4}{10}$

$=$

6. Let the life span of a light bulb be given by the pdf $f(x) = ce^{-x/3}$ where $x > 0$.

(a) Compute c

(b) Compute $P(X < 2)$.

(a) $\int_0^\infty f(x)\,dx = 1$ $\quad \int_0^\infty c e^{-x/3}\,dx = 1$

So $-3ce^{-x/3}\Big|_0^\infty = 1 \Rightarrow -3\cancel{c}(e^{-\infty}) - -3ce^0 = 1$

0

$3c = 1$

$\boxed{c = \frac{1}{3}}$

(b) $P(X<2)$

$= \int_0^2 f(x)\,dx = \int_0^2 \frac{1}{3}e^{-x/3}\,dx$

$= -e^{-x/3}\Big|_0^2 = \boxed{-e^{-2/3} + 1}$