3.5 : 1-8, 11, 14, 15, 17, 19, 20, 21

(1) $f(x) = \begin{cases} 630\, x^4 (1-x)^4 & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$

(a) $P(0.2 < X < 0.8)$

FIRST FIND $\mu = E(X) = \int_0^1 x \cdot f(x)\, dx = \int x \cdot 630\, x^4 (1-x)^4\, dx$

$= 630 \int_0^1 x^5 (1-x)^4 dx = 630 \int_0^1 x^5 (1 - 4x + 6x^2 - 4x^3 + x^4)\, dx$

$= 0.50$

AND

$VAR(X) = \int_0^1 (x - .5)^2 f(x)\, dx = \frac{3}{11} - \frac{1}{4} = \frac{1}{44}$

$\sigma^2 = \frac{1}{44}$

$\sigma = \sqrt{\frac{1}{44}}$

$= P(0.2 - .5 < X - .5 < 0.8 - .5) = P(|X - .5| < 0.3)$

$P(|X - \mu| < k\sigma)$

$> 1 - \frac{1}{k^2} = 1 - \frac{1}{(0.3\sqrt{44})^2}$

$= .7475$

so $k\sigma = 0.3$

$\Rightarrow k = \frac{0.3}{\sigma} = \frac{0.3}{\sqrt{\frac{1}{44}}} = 1.9899$

THUS

$P(0.2 < X < 0.5) > .7475$

(b) $P(0.2 < X < .8)$

$\int_{0.2}^{0.8} f(x)\, dx \xrightarrow{\text{used techology}} = .9608$

§ 3.5 1 - 8, 11, 14, 15, 17, 19, 20, 21

(2.) $\lambda = 100$ Poisson $\xrightarrow{\text{For Poisson we know}}$ $\mu = \lambda$, $\sigma^2 = \lambda$ so $\mu = 100$, $\sigma^2 = 100$

$$P(70 < P < 130)$$

$$P(|P - 100| < 30) \geq 1 - \frac{1}{k^2} = 1 - \frac{1}{9} = \frac{8}{9}.$$

BY CHEBYSHEV

$k\sigma = 30$

$$k = \frac{30}{\sigma} = \frac{30}{10} = 3$$

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(3) YOU CAN SKIP THIS .. BUT FOR YOUR ENJOYMENT

$$\sigma^2 = \sum (x-\mu)^2 p(x)$$

$$\sigma^2 = \sum_{x-\mu < -k\sigma} (x-\mu)^2 p(x) + \sum_{|x-\mu| < k\sigma} (x-\mu)^2 p(x) + \sum_{x-\mu > k\sigma} (x-\mu)^2 p(x)$$

$$\sigma^2 \geq \sum_{x-\mu < -k\sigma} (x-\mu)^2 p(x) + 0 + \sum_{x-\mu > k\sigma} (x-\mu)^2 p(x)$$

$$\geq \sum_{x-\mu < -k\sigma} (-k\sigma)^2 p(x) \qquad \sum_{x-\mu > k\sigma} (k\sigma) p(x)$$

$$= k^2\sigma^2 \left[\sum_{x-\mu < -k\sigma} p(x) + \sum_{x-\mu > k\sigma} p(x) \right]$$

so

$$\sigma^2 \geq k^2\sigma^2 \left[P(x-\mu < -k\sigma) + P(x-\mu > k\sigma) \right] \quad \div k\sigma^2$$

$$\frac{1}{k^2} \geq P(x-\mu < -k\sigma) + P(x-\mu > k\sigma)$$

THUS

$$P(|x-\mu| < k\sigma) \geq 1 - \frac{1}{k^2}$$

(4) POISSON $\lambda = 120$ $\xrightarrow[\text{as before}]{\text{so}}$ $\mu = 120$, $\sigma^2 = 120$

$$P(100 < X < 140) = P(|x - 120| < 20) \geq 1 - \frac{1}{k^2}$$

$k\sigma = 20$

$k = \frac{20}{\sqrt{120}}$

$$= 1 - \frac{1}{\left(\frac{20}{\sqrt{120}}\right)^2}$$

$$= \boxed{\frac{8}{20}}$$

(5.) FIND SMALLEST VALUE OF n in a binomial distribution for which

$$P\left(\left|\frac{X_n}{n} - p\right| < 0.1\right) \geq 0.90$$

LAW OF LARGE NUMBERS

$$P(|\bar{X} - \mu| < c) \geq 1 - \frac{\sigma^2}{n \cdot c^2}$$

SO $c = .1$

AND $1 - \frac{\sigma^2}{n c^2} = .90$

$$1 - \frac{\sigma^2}{n(.1)^2} = .90$$

SOLVE FOR $\boxed{n = 10^3 \sigma^2}$

3.5.6 HOW LARGE SHOULD A SAMPLE BE SO THAT

$$P\left(|\bar{X}-\mu| < \frac{\sigma}{2}\right) \geq 90\%$$

L of L NUMBERS

$$P(|\bar{X}-\mu| < c) \geq 1 - \frac{\sigma^2}{nc^2}$$

so

$c = \frac{\sigma}{2}$

$$1 - \frac{\sigma^2}{nc^2} = .90$$

$$1 - \frac{\sigma^2}{n\left(\frac{\sigma}{2}\right)^2} = .90$$

$$\frac{\sigma^2}{n\frac{\sigma^2}{2^2}} = .1 \implies \boxed{n = 40}$$

3.5.7

$\frac{S_n}{n}$ will be "near" p for LARGE n.

Note $p \cdot q \geq \frac{1}{4}$, $VAR\left(\frac{S_n}{n}\right) = pq$ what else can you say?

3.5.8 Typo in problem!

11 14 15 17 19 20 21

3.5.11 $X_1, \ , X_n$ iid with

$$f(x_i) = \begin{cases} 0.2 & x_i = 0, 1, 2, 3, 4 \\ 0 & \text{otherwise} \end{cases}$$

USING CTL FIND APPROX VALUE OF

$P(\overline{X}_{100} > 2)$

SOL'N

x	0	1	2	3	4	
$f(x)$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$E(x) = 2$
$xf(x)$	0	$\frac{1}{5}$	$2/5$	$3/5$	$4/5$	$\xrightarrow{\text{SUM UP}}$ $\frac{10}{5} = 2$ $\mu = 2$
$x^2 f(x)$	0	$\frac{1}{5}$	$4/5$	$9/5$	$16/5$	$\xrightarrow{\text{SUM UP}}$ $E(x^2) = \frac{30}{5} = 6$

So $VAR(x) = E(x^2) - E(x)^2 = 6 - 2^2$
$= 2$

So $\mu_X = 2$

AND $\sigma_X = 2$

THUS $\mu_{\bar{x}} = 2$

$\sigma_{\bar{x}} = \frac{2}{\sqrt{n}} = \frac{2}{\sqrt{100}}$ $\longrightarrow$ USE NORMAL WITH $\mu = 2$ & $\sigma_{\bar{x}} = \frac{2}{\sqrt{100}}$

So $\boxed{P(\bar{x}_{100} > 2) = .50}$