3.3 : 1, 3, 5, 6, 7, 9, 15, 16

(1.) BIN WITH 8 operable, 6 def, 10 semi-operable.

We draw 4.

X = COUNT OF OPERABLE , Y = COUNT OF def.

	Y = 0	1	2	3	4
X = 0	1.98%				
1	6.77%	20.3% YOU FILL IN THE REST.			0
2	6.35%			0	
3	1.88%		0	0	0
4	.14%	0	0	0	0

(a) $P(X=0, Y=0) = \dfrac{{}_6C_0 \; {}_8C_0 \; {}_{10}C_4}{{}_{24}C_4}$, $P(X=2, Y=1) = \dfrac{{}_6C_2 \; {}_8C_1 \; {}_{10}C_1}{{}_{24}C_4}$

so $f(x,y) = \dfrac{{}_6C_x \cdot {}_8C_y \cdot {}_{10}C_{(4-x-y)}}{{}_{24}C_4}$ where $0 \le x+y \le 4$

(b) $P(X=3, Y=0)$

$= \dfrac{{}_6C_3 \cdot {}_8C_0 \cdot {}_{10}C_1}{{}_{24}C_4} = 1.88\%$

(c) $P(X<3, Y=1) = f(0,1) + f(1,1) + f(2,1)$

$= 40.65\%$

(d) for you...

③ $f(x,y) = C(1-x)(1-y) \quad -1 \le x \le 1, \; -1 \le y \le 1$

FIND c.

$$\int_{-1}^{1}\int_{-1}^{1} f(x,y)\,dx\,dy = \int_{-1}^{1}\int_{-1}^{1} c(1-x)(1-y)\,dx\,dy = 1$$

$$\int_{-1}^{1} c(1-y)\underbrace{\left[x - \frac{x^2}{2}\right]_{-1}^{+1}} dy = 1$$

$$\left(\left[1-\tfrac{1}{2}\right] - \left[-1-\tfrac{1}{2}\right]\right) = 2$$

$$\int_{-1}^{1} c\,\underbrace{(1-y)}\cdot 2\cdot dy = 1$$

$$2c\cdot\left(y - \frac{y^2}{2}\right)\Big|_{-1}^{+1} = 1$$

$$2c\left[\left(1-\tfrac{1}{2}\right) - \left(-1-\tfrac{1}{2}\right)\right] = 1$$

$$4c = 1 \Rightarrow \boxed{c = \tfrac{1}{4}}$$

(5.)

				y	
		-2	0	1	4
	-1	0.3	0.1	0.0	0.2
x	3	0.0	0.2	0.1	0.0
	5	0.1	0.0	0.0	0.0

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y	-2	0	1	4
f_y	.4	.3	.1	.2

x	-1	3	5
f_x	.6	.3	.1

(6.) $f(x,y) = \frac{1}{5}(3x - y), \quad 1 \le x \le 2 \; ; \quad 1 \le y \le 3$

$$f_x(x) = \int_1^3 f(x,y)\,dy = \frac{1}{5}\int 3x - y\,dy$$

$$= \frac{1}{5}\left[3xy - \frac{y^2}{2}\right]_{y=1}^{y=3} = \frac{1}{5}\left[\left(9x - \frac{9}{2}\right) - \left(3x - \frac{1}{2}\right)\right]$$

$$\boxed{f_x(x) = \frac{1}{5}\left[6x - 4\right]}$$

$$f_y(y) = \int_1^2 f(x,y)\,dx = \int_1^2 \frac{1}{5}(3x - y)\,dx$$

$$= \frac{1}{5}\left[\frac{3x^2}{2} - yx\right]_1^2 = \frac{1}{5}\left[\left(\frac{3(2)^2}{2} - y(2)\right) - \left(\frac{3(1)^2}{2} - y\right)\right]$$

$$\boxed{f_y(y) = \frac{1}{5}\left[\frac{9}{2} - y\right]}$$

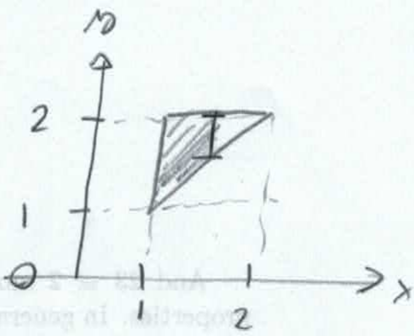
(7.) $P(X = -1 \mid Y = 0) = \frac{P(X = -1 \cap Y = 0)}{P(Y = 0)}$

$$= \frac{f(-1,0)}{f(-1,0) + f(3,0) + f(5,0)} = \frac{.1}{.1 + .2 + 0} = \frac{1}{3}$$

OR we could use $f_y(0) = .3$

3.3

(9.) $f(x,y) = \frac{8}{9}xy \qquad 1 \le x \le y \le 2$



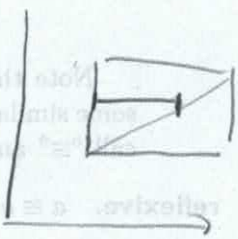
$$f_x(x) = \int_x^2 \frac{8}{9}xy \, dy$$

$$= \frac{8}{9}x \int_x^2 y\,dy = \frac{8}{9}x \left.\frac{y^2}{2}\right|_x^2$$

$$= \frac{\cancel{8}^{\,4}}{9} x \cdot \frac{1}{\cancel{2}} \left[2^2 - x^2\right] = \frac{4}{9}x\left[4 - x^2\right]$$

$$f_y(y) = \int_1^y \frac{8}{9}xy \, dx$$

$$= \frac{\cancel{8}^{\,4}}{9} y \left.\frac{x^2}{\cancel{2}}\right|_1^y = \boxed{\frac{4}{9}y\left[y^2 - 1\right]}$$



15 c) CONTINUED

X	1	2	3
f_X	$\frac{3}{6}$	$\frac{4}{12}$	$\frac{2}{12}$
$x \cdot f$	$\frac{3}{6}$	$\frac{8}{12}$	$\frac{6}{12}$
$x^2 f$	$\frac{3}{6}$	$\frac{16}{12}$	$\frac{18}{12}$

$\xrightarrow{\text{SUM UP}}$ $E(x) = \frac{20}{12}$ $\quad \mu = \frac{20}{12}$

$\xrightarrow{\text{SUM UP}}$ $E(x^2) = \frac{40}{12}$

$VAR(x) = E(x^2) - (E(x))^2$

$= \frac{40}{12} - \left(\frac{20}{12}\right)^2$

$\boxed{VAR(x) = \frac{5}{9}}$

SIMILARLY FOR Y

Y	1	2	3
$f_Y(y)$	$5/12$	$4/12$	$3/12$
$y f_Y(y)$	$\frac{5}{12}$	$\frac{8}{12}$	$\frac{9}{12}$
$y^2 f_Y(y)$	$\frac{5}{12}$	$\frac{16}{12}$	$\frac{27}{12}$

$\xrightarrow{\text{SUM UP}}$ $\frac{22}{12} = \frac{11}{6}$ $\quad E(Y) = 11/6$

$\xrightarrow{\text{SUM UP}}$ $\frac{48}{12} = 4$ $\quad E(Y^2) = 4$

So $VAR(Y) = E(Y^2) - (E(Y))^2$

$= 4 - \left(\frac{11}{6}\right)^2$

So $= \frac{144-121}{36} = \boxed{\frac{23}{36}}$

$$\rho_{XY} = \frac{Cov.(X,Y)}{\sqrt{VAR\,X}\,\sqrt{VAR\,Y}} = \frac{-5/36}{\sqrt{5/9}\cdot\sqrt{23/36}} = -0.23$$

3.3.15

X \ Y	1	2	3
1	1/6	1/6	1/6
2	1/6	1/12	1/12
3	1/12	1/12	0

(a) FIND $E(XY)$

$$= \boxed{1\cdot 1\cdot \tfrac{1}{6}} + 1\cdot 2\cdot \tfrac{1}{6} + 1\cdot 3\cdot \tfrac{1}{6}$$

$$+ \cdots \quad 3\cdot 3\cdot 0$$

when $x=1$ & $y=1$

$x\,y\,f(x,y)$

$1\cdot 1\; f(1,1) = 1\cdot 1\cdot \tfrac{1}{6}$

$$= \frac{2+4+6+4+4+6+3+6+0}{12}$$

$$= \frac{35}{12},$$

COMPUTE

(b) $\boxed{\mathrm{COV}(X,Y) = E(XY) - E(X)\cdot E(Y)}$

$$E(X) = (1)\cdot\left(\frac{3}{6}\right) + 2\cdot\left(\frac{4}{12}\right) + 3\left(\frac{2}{12}\right)$$

$$= \frac{6+8+6}{12} = \frac{20}{12}$$

$$E(Y) = 1\left(\frac{5}{12}\right) + 2\left(\frac{4}{12}\right) + 3\left(\frac{3}{12}\right) = \frac{5+8+9}{12} = \frac{22}{12}$$

So $$\mathrm{COV}(X,Y) = \frac{35}{12} - \frac{20}{12}\cdot\frac{22}{12} = \frac{35\cdot 12 - 20\cdot 22}{12\cdot 12} = \frac{-20}{144} = \boxed{\frac{-5}{36}}$$

(c) $$\rho_{X,Y} = \frac{\mathrm{COV}(X,Y)}{\sqrt{\mathrm{VAR}(X)}\cdot\sqrt{\mathrm{VAR}(Y)}}$$

(c)

$$E(X^2) = \int_0^2 \int_0^2 x^2 f(x,y)\, dx\, dy = \cdots = \frac{12}{7}$$

$$E(Y^2) = \int_0^2 \int_0^2 y^2 f(x,y)\, dx\, dy = \cdots = \frac{32}{21}$$

So $\quad VAR(X) = E(X^2) - E(X)^2 = \frac{12}{7} - \left(\frac{25}{21}\right)^2 = \frac{131}{441}$

and $\quad VAR(Y) \; E(Y^2) - E(Y)^2 = \frac{32}{21} - \left(\frac{23}{21}\right)^2 = \frac{143}{441}$

So $\quad \rho_{X,Y} = \frac{COV(X,Y)}{\sqrt{VAR(X)} \cdot \sqrt{VAR(Y)}} = \frac{-8/441}{\sqrt{\frac{131}{441}} \cdot \sqrt{\frac{143}{441}}} = \frac{-8}{\sqrt{131 \cdot 143}} \approx -0.058$

$\boxed{3.3.16}$ $f(x,y) = \frac{1}{28}(4x+2y+1)$ $0 \le x \le 2,\ 0 \le y \le 2$

(a) FIND $E(XY)$ | (b) $\mathrm{Cov}(X,Y)$ | (c) $\rho_{X,Y}$

(a) $$E(XY) = \int_0^2\int_0^2 xy\, f(x,y)\,dx\,dy$$

$$= \int_0^2\int_0^2 x\cdot y\,\frac{1}{28}(4x+2y+1)\,dx\,dy$$

$$= \frac{1}{28}\int_0^2\int_0^2 4x^2y + 2xy^2 + xy\,dx\,dy$$

$$= \frac{1}{28}\int_0^2 \left[\frac{4x^3}{3}y + x^2y^2 + \frac{x^2}{2}y\right]_0^2 dy = \frac{1}{28}\int_0^2 \frac{32}{3}y + 4y^2 + 2y\,dy$$

$$= \frac{1}{28}\int_0^2 \left[\frac{38}{3}y + 4y^2\right]dy = \frac{1}{28}\left[\frac{19}{3}y^2 + \frac{4}{3}y^3\right]_0^2 = \frac{1}{28}\left[\frac{76}{3} + \frac{32}{3}\right]$$

$$= \frac{1}{\cancel{28}_{\,7}}\,\frac{\cancel{108}^{\,27}}{3} = \frac{27}{21} = \boxed{\frac{9}{7}}$$

(b) $$\mathrm{Cov}(X,Y) = E(XY) - E(X)E(Y)$$

$$E(X) = \int_0^2\int_0^2 x\,f(x,y)\,dx\,dy = \dots = \frac{25}{21}$$

AND

$$E(Y) = \int_0^2\int_0^2 y\,f(x,y)\,dx\,dy = \dots = \frac{23}{21}$$

So $$\mathrm{Cov}(X,Y) = \frac{9}{7} - \left(\frac{25}{21}\right)\left(\frac{23}{21}\right)$$

$$= \frac{-8}{441}$$