3.2 1, 2, 3-9 odd, 10, 15, 17, 21, 22, 27

①

(a) $P(X=7) = \binom{10}{7}\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right)^3 = \frac{10!}{7!\,3!}\left(\frac{1}{2}\right)^{10} = \frac{\overset{5\ 3}{\cancel{10}\cdot\cancel{9}\cdot\cancel{8}}}{1\cdot\cancel{2}\cdot\cancel{3}}\cdot\frac{1}{2^{\cancel{10}\,7}} = \frac{15}{2^7} = \frac{15}{128}$

(b) $P(X\le 7) = 1 - P(X>7) = 1 - \left[P(X=8) + P(X=9) + P(X=10)\right]$

BINOMIAL $p(x) = \binom{n}{x}\cdot p^x q^{n-x}$

$= 1 - \left[\binom{10}{8}\left(\frac{1}{2}\right)^8\left(\frac{1}{2}\right)^2 + \binom{10}{9}\left(\frac{1}{2}\right)^9\left(\frac{1}{2}\right)^1 \binom{10}{10}\left(\frac{1}{2}\right)^{10}\left(\frac{1}{2}\right)^0\right] = \cdots$

(c) $P(X>0) = 1 - P(X=0) = 1 - \binom{10}{0}\left(\frac{1}{2}\right)^0\cdot\left(\frac{1}{2}\right)^{10} = \frac{1023}{1024}$

(d) $\mathbb{E}(X)$ AND $\mathrm{VAR}(X)$

$\mathbb{E}(X) = np = 10\left(\frac{1}{2}\right) = 5$ $\qquad \mathrm{VAR}(X) = npq = 10\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = 2.5$

---

②

(a) $P(X=0) = \frac{e^{-3}(3)^0}{0!} = e^{-3}$

(b) $P(X\ge 4) = 1 - P(X<4)$

$= 1 - \left[\frac{e^{-3}(3)^0}{0!} + \frac{e^{-3}(3)^1}{1!} + \frac{e^{-3}(3)^2}{2!} + \frac{e^{-3}(3)^3}{3!}\right] = \cdots$

POISSON

$f(x) = \frac{e^{-\lambda}\cdot\lambda^x}{x!}$ $\quad x = 0, 1, 2, \ldots$

λ = average

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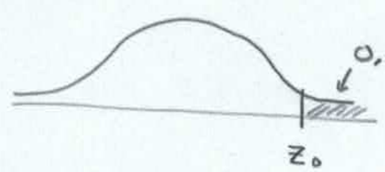
③ (a) $P(Z > z_0) = 0.05$

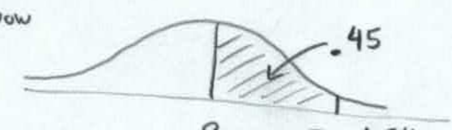
So this

$z_0 = 1.54$

WE WANT



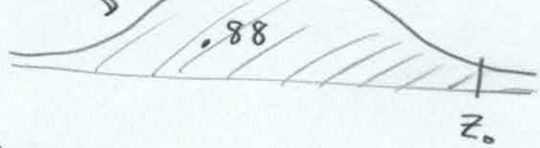
WE KNOW



FROM TABLE

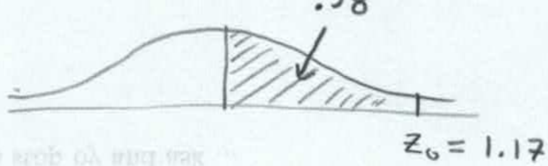
(3.) (b) $P(Z < z_0) = 0.88$

we WANT



$\boxed{z_0 = 1.17}$

we can find



(c) $P(Z < z_0) = .10$

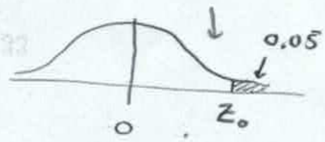
$\boxed{z_0 = -1.28}$

(d) $P(Z > z_0) = .95$

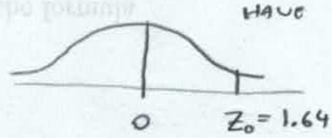
$\boxed{z_0 = -1.64}$

(4.) $\boxed{X \sim N(12, 5)}$

(a) $P(X > x_0) = 0.05$ $\quad\Big\|\quad$ $P(Z > z_0) = 0.05$



$z_0 = 1.64 \longrightarrow$ FROM CHART WE HAVE



$x_0 = \mu + \sigma \cdot z_0 = 12 + 5(1.64)$

$= 12 + 8.2 = 20.2$

(b) $P(x < x_0) = .98$ $\qquad$ $z_0 = 2.06$ $\quad (P(Z < 2.06) = .98)$

FROM CHART

$x_0 = \mu + \sigma z_0$

$= 12 + 5(2.06) = 12 + 10.3 = 22.3$

(c) $P(X < x_0) = 0.20$ $\qquad$ $z_0 = .84$

$x_0 = \mu + \sigma z_0$

$= 12 + 5(.84) = 12 + 4.2 = 16.2$

④(d) $P(X > x_0) = 0.90$

$P(Z > z_0) = .90$

$z_0 = 1.28$

$X_0 = \mu + \sigma z_0$

$= 12 + 5(1.28) = 12 + 6.4 = 18.4$

5. $X \sim N(10, 25)$.

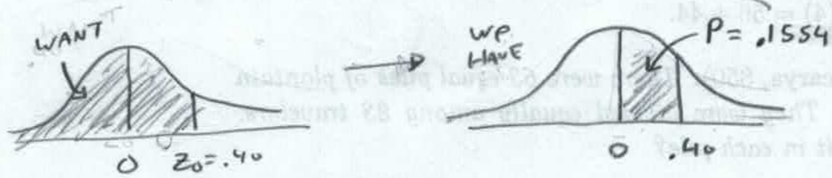
(a) $P(X \le 20)$

$$Z = \frac{X-\mu}{\sigma} = \frac{20-10}{25} = \frac{10}{25} = .40$$

$P(Z \le .40) = 0.5 + .1554 = \boxed{.6554}$

So

$$\boxed{\begin{aligned} &P(X \le 20) \\ &= P(Z \le .40) \\ &= .6554 \end{aligned}}$$



(b) $P(X > 5)$

$$Z_0 = \frac{5-10}{25} = \frac{-5}{25} = -0.20$$

$\boxed{P(X > 5) = P(Z > -.20) = .5793}$ FROM CHART

(c) $P(12 < X < 15)$

$$Z_1 = \frac{X_1-\mu}{\sigma} = \frac{12-10}{25} = .08 \quad \Big| \quad Z_2 = \frac{X_2-\mu}{\sigma} = \frac{15-10}{25} = .20$$

So $\boxed{P(12 < X < 15) = P(.08 < Z < .20) = .0474}$

$P(0 \le Z \le .20) = .0793$

$P(0 \le Z \le .08) = .0319$

$.0793 - .0319 = .0474$

(5.) (d) $P(|X-12| \le 15)$

$= P(-15 \le X-12 \le 15)$

$= P(-3 \le X \le 27)$

$z_2 = \frac{27-10}{25} = \frac{17}{25} = .68$

$z_1 = \frac{-3-\mu}{\sigma} = \frac{-3-10}{25} = \frac{-13}{25} = -.52$

So $P(|X-12| \le 15) = P(-.52 \le Z \le .68)$

(6) Binomial with $p = .62$ with $n = 16$.

(a) $p(12) = \binom{16}{12} p^{12} \cdot q^{4} = \frac{16!}{12!4!}(.62)^{12}(.38)^4 = 12.2\%$

$16-12=4$

(b) $p(X>8) = \binom{16}{9} p^9 q^7 + \binom{16}{10} p^{10} \cdot q^6 + \binom{16}{11} p^{11} \cdot q^5$

$+ \cdots + \binom{16}{16} p^{16} \cdot q^4$

$= 77.0\%$

(c) 77 out of 100 games he will complete more than half of his passes.

(d) $E(B) = n \cdot p = 16(.62)$

(7.) $B(.70, 15)$

(a) $P(B=10) = \binom{15}{10} (p)^{10} (q)^5 = \binom{15}{10} (.70)^{10} (.30)^5 = 20.6\%$

(b) $P(B \le 10) = \binom{15}{0} p^0 q^{15} + \cdots + \binom{15}{10} p^{10} \cdot q^5$

$= 0.485$

(c) $E(B) = \mu_B = n \cdot p = (15)(.70) = 10.5$

$\boxed{3.2.8}$ BINOMIAL $p = .40$ $n = 6$

(a) $P(B \geq 1) = 1 - P(B < 0)$

$$= 1 - \binom{6}{0} p^0 q^6 = 1 - \frac{6!}{6!0!} (.40)^0 \cdot (.60)^6$$

$$= 95.3\%$$

(b) PROB OF FIRING ONCE & HITTING AT LEAST ONCE $= .40$

— " TWICE $= 1 - \underbrace{(.60)^2}_{\text{PROB OF MISSING TWICE}} = 1 - .36 = .72$

— " THRICE $= 1 - (.6)^3 = 1 - .216$

$= .784$

$\boxed{3 - \text{TIMES}}$

(c)

9. BINOMIAL $p = \frac{3}{100}$ ← defective, $n = 400$

(a) $P(X = r) = \binom{400}{r} (p)^r (q)^{400-r}$

$$= \binom{400}{r} (.03)^r (.97)^{400-r}$$

(b) $P(X \geq k) = \binom{400}{k} (.03)^k (.97)^{400-k} + \binom{400}{k+1} (.03)^{k+1} (.97)^{400-(k+1)}$

$+ \cdots + \binom{400}{400} (.03)^{400} (.97)^0$

(c) $P(X = 0 \text{ or } X = 1) = \binom{400}{0} (.03)^0 (.97)^{400} + \binom{400}{1} (.97)^1 (.97)^{399}$

$= 6.84 \times 10^{-5} = 0.0000684 = 0.00684\%$

3.2

(10) $\lambda = \frac{1}{2}$ POISSON $P(x) = \dfrac{e^{-\lambda} \cdot \lambda^{x}}{x!} = \dfrac{e^{-1/2} \cdot (\frac{1}{2})^{x}}{x!}$

(a) $P(X \geq 1) = 1 - P(X=0) = 1 - \dfrac{e^{-1/2} (\frac{1}{2})^{0}}{0!} = 1 - e^{-1/2}$

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(15) X is uniform continuous $a=0,\ b=100$ $\Big] \rightarrow$ so $f(x) = \dfrac{1}{b-a} = \dfrac{1}{100}$

(a) $P(60 \leq X \leq 80) = \displaystyle\int_{60}^{80} \frac{1}{100}\, dx$

$= \dfrac{1}{100} x \Big|_{60}^{80} = 0.20$

(b) $P(X \geq 90) = \displaystyle\int_{90}^{100} \frac{1}{100}\, dx = \frac{1}{100} x \Big|_{90}^{100} = \frac{1}{100}[100 - 90] = 0.10$

(c) for you...

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(17) $\dfrac{1}{\beta} = 0.05 \rightarrow$ Exponential $f(x) = \dfrac{1}{\beta} e^{-x/\beta}$ $x \geq 0$

$P(0 \leq X \leq 10) = \displaystyle\int_{0}^{10} f(x)\, dx = \int_{0}^{10} \frac{1}{\beta} e^{-x/\beta}\, dx$ so $f(x) = 0.05\, e^{-0.05x}$

$= \displaystyle\int_{0}^{10} 0.05\, e^{-0.05x}\, dx = -e^{-0.05x} \Big|_{0}^{10}$

$= -e^{(-0.05)(10)} - \left(-e^{(-0.05)(0)}\right) = \boxed{-e^{-1/2} + 1}$

(20.) We can skip for now.

(21.) $N(\underset{\mu}{72}, \underset{\sigma}{6})$

FIND n_0, the score, so that $P(N > n_0) = .15$

<u>SOL'N</u>: FIRST FIND Z-SCORE FROM TABLE

$$\left.\begin{array}{l} P(Z > 1.04) = .15 \\ \text{so } Z_0 = 1.04 \end{array}\right] \longrightarrow \text{THUS } n_0 = \mu + \sigma z_0$$

$$= 72 + 6(1.04) = 72 + 6.24$$

$$= 78.24$$