Math 6250: Notes

1 Sequences in Metric Spaces

Definition 1.1. *Let (X, d) be a metric space. And let $(x_n)_{n=1}^{\infty}$ be a sequence in X. We say (x_n) is* **Cauchy** *if for all $\varepsilon > 0$ there exists a $N \in \mathbb{N}$ so that $n, m > N$ implies $d(x_n, x_m) < \varepsilon$.*

Definition 1.2. *Let (X, d) be a metric space. And let $(x_n)_{n=1}^{\infty}$ be a sequence in X. We say (x_n)* **converges** *to L in X if for all $\varepsilon > 0$ there exists a $N \in \mathbb{N}$ so that $n > N$ implies $d(x_n, L) < \varepsilon$. We write $x_n \to L$. Or equivalently if (x_n)* **converges** *to L in X if $\lim_{n\to\infty} d(x_n, L) = 0$ in the usual metric.*

Definition 1.3. *Let (X, d) be a metric space. And let $f : x \to X$. We say f is a* **contraction** *if there is some $M \in \mathbb{R}$ $M \in (0, 1)$ so that*

$$d(f(x), f(y)) < Md(x, y)$$

$n > N$ implies $d(x_n, L) < \varepsilon$. We write $x_n \to L$. for all $x, y \in X$.

Theorem 1.4. *Let (X, d) be a metric space. X is complete if and only if for all (x_n) that are Cauchy $X_n \to L$ where $L \in X$.*