Math 6250 Quiz 2

Name:____

1. The following are related:
   (a) Let $A, B \subseteq \mathbb{R}$ show if $A \subseteq B$ then $\inf(A) \geq \inf(B)$ and $\sup(A) \leq \sup(B)$.
   (b) Let (X, d) be a metric space and let $x \in X$ and $A, B \subseteq X$. Assume $A \subseteq B$. Show $\text{dist}(x, A) \geq \text{dist}(x, B)$.
   (c) Let (X, d) be a metric space and let $x \in X$ and $A, B \subseteq X$. Show $\text{dist}(x, A \cup B) = \inf(\text{dist}(x, A), \text{dist}(x, B))$.

2. Compute the following distances.
   (a) Consider the metric space $(\mathbb{N}, d)$ where $d(n_1, n_2) = |\frac{1}{n_1} - \frac{1}{n_2}|$. Compute $d(z, S)$ where $z = 17$ is any point in $\mathbb{R}$ and S is the set of evens.
   (b) Consider the metric space $(X, d) = (\mathbb{R}, e)$ where e is the Euclidean metric. Compute $d(z, S)$ where z is any point in $\mathbb{R}$ and $S = \mathbb{Q}$.
   (c) Consider the metric space $(X, d) = (\mathbb{R}^2, e)$ where e is the Euclidean metric. Compute $d(z, S)$ where $z = (0, 0)$ and $S = \{(x, y) : y = \frac{2}{x}\}$.
   (d) Consider the metric space $(C[0, 1], d)$ where $d(f, g) = \int_0^1 |f - g| dx$. Compute $d(f, S)$ where $f(x) = 0$ and $S = \{\sin nx : n \in \mathbb{N}\}$.

3. Compute the acc(S) and iso(S) for each of the following:
   (a) Consider the metric space $(\mathbb{N}, d)$ where $d(n_1, n_2) = |\frac{1}{n_1} - \frac{1}{n_2}|$. Let S is the set of evens.
   (b) Consider the metric space $(X, d) = (\mathbb{R}, e)$ where e is the Euclidean metric. Let $S = (0, 1)$.
   (c) Consider the metric space $(X, d) = (\mathbb{R}, e)$ where e is the Euclidean metric. Let $S = \mathbb{Q}$.
   (d) Consider the metric space $(X, d) = (\mathbb{R}^2, e)$ where e is the Euclidean metric. Let $S = \{(x, y) : y \geq \frac{2}{x} \text{ and } x > 0\}$.
   (e) Consider the metric space $(C[0, 1], d)$ where $d(f, g) = \int_0^1 |f - g| dx$. Let $S = \{\sin x/n : n \in \mathbb{N}\}$.

4. Do one of the following:
   (a) Let (X, d) be a metric space and let $S \subseteq X$. Show that
   $$z \in \text{acc}(S) \iff \text{For all } \varepsilon > 0, D \setminus \{z\} \cap S \neq \emptyset.$$
   where $D = \{x \in X : d(z, x) < \varepsilon\}$.
   (b) Let (X, d) be a metric space and let $S \subseteq X$. Show that
   $$z \in \text{iso}(S) \iff \text{there exists } \varepsilon > 0 \text{ so that } D \setminus \{z\} \cap S = \emptyset$$
   where $D = \{x \in X : d(z, x) < \varepsilon\}$.

5. Do problems 1,4,5,6 from section 2.