Math 6250 Quiz 1

Name:____

1. Define $\mathcal{P}_2 = \{a_2x^2 + a_1x^1 + a_0 : a_i \in \mathbb{R}\}$ and the function $d : \mathcal{P}_2 \times \mathcal{P}_2 \to \mathbb{R}$ given by
$$d(a_2x^2 + a_1x^1 + a_0, b_2x^2 + b_1x^1 + b_0) = |a_2 - b_2| + |a_1 - b_1| + |a_0 - b_0|.$$
Show $(\mathcal{P}_2, d)$ is a metric space.

2. Let (X, d) and (X, e) be two metric spaces. Prove
   (a) that (X, f) is not necessarily a metric space where $f : X \times X \to \mathbb{R}$ given by
   $$f(a, b) = \min\{d(a, b), e(a, b)\}.$$
   (Q1.6)
   (b) And that (X, g) is a metric space where $g : X \times X \to \mathbb{R}$ given by
   $$g(a, b) = \max\{d(a, b), e(a, b)\}.$$

3. Set $d(m, n) = |m^{-1} - n^{-1}|$, $d(n, \infty) = d(\infty, n) = n^{-1}$ and $d(\infty, \infty) = 0$ for all $m, n \in \mathbb{N}$. Show that d is a metric on $\widetilde{\mathbb{N}} = \mathbb{N} \cup \{\infty\}$. (Q1.14).

4. Let $f : \mathbb{R} \to \mathbb{R}$ be an increasing subadditive function so that $f(0) = 0$. Let (X, d) be a metric space. A function is *subadditive* if $f(a + b) \leq f(a) + f(b)$. Is $f(d)$ a metric? Prove or disprove.

5. Suppose (X, d) is a metric space, $z \in X$ and $k \in \mathbb{R}^+$. Define $v : X \to \mathbb{R}$ by $v(x) = \delta_z(x) + k$. Show v is pointlike. (Q1.17)

6. Show isometries are an equivalence relation. (Q1.23)