Math 2320 - Practice Final Exam

1 Integrals

1. $\int x^2\sqrt{3-4x^3}\,dx$ - try u-sub
2. $\int e^{3x}\,dx$
3. $\int \dfrac{x^2}{3-4x^3}\,dx$
4. $\int \dfrac{x^2}{1+(3-4x^3)^2}\,dx$
5. $\int \dfrac{x\sin(x^2)}{\cos(x^2)}\,dx$
6. $\int x^2e^{3x}\,dx$ - try by parts
7. $\int x\sin(4x)\,dx$
8. $\int \ln(x)\,dx$
9. $\int x^2\ln(x)\,dx$
10. $\int \sec^3(x)\,dx$ - I used $u=\sec(x)$ and $dv=\sec^2(x)$
11. $\int \sin^3(x)\cos(x)\,dx$ - trigonometric integral
12. $\int \sin^3(x)\cos^2(x)\,dx$
13. $\int \sin^{1/2}(x)\cos^3(x)\,dx$
14. $\int \sin^2(3x)\cos^2(3x)\,dx$

15. $\int \sin^4(2x)\,dx$

16. $\int \frac{1}{\sqrt{1-x^2}}\,dx$ - trigonometric substitution

17. $\int \frac{1}{\sqrt{x^2-1}}\,dx$

18. $\int \sqrt{1+x^2}\,dx$ Use the result from 10

19. $\int \frac{1}{\sqrt{1+x^2}}\,dx$

20. $\int \frac{2-x^2}{x^2(1+x^2)}\,dx$ - try partial fractions

21. $\int \frac{6x^2-4}{x(x^2-1)}\,dx$

22. $\int \frac{3x^3+x+1}{x^3-x}\,dx$

2 Applications of Integration

1. Find the area between the functions $y = x^2$ and $y = 4$.

2. Find the volume by revolving the region R around the x-axis where R is in the first quadrant defined by $y = x^2$, $x = 9$ and $y = 0$.

3. Find the volume by revolving the region R around the y-axis where R is in the first quadrant defined by $y = x^2$, $x = 9$ and $y = 0$.

4. Find the length of the curve $y = 1 + 3x$ from the point $(0, 1)$ to the point $(2, 7)$.

3 Sequences and Series

1. Compute the following limits

   (a) $\lim_{n\to\infty} \frac{n^3-1}{1-7n^3}$

(b) $\lim_{n\to\infty} \dfrac{n^2-1}{1-7n^3}$

(c) $\lim_{n\to\infty} \dfrac{n^3-1}{1-7n^2}$

(d) $\lim_{x\to 0} \dfrac{\sin(x)}{x}$ - Try L'Hopital's Rule

(e) $\lim_{x\to 0} \dfrac{1-e^{x^2}}{x^2}$

(f) $\lim_{x\to 0} \dfrac{1}{x}$ - I think it is $+\infty$, $-\infty$ or DNE.

(g) $\lim_{x\to 0} \dfrac{1}{x^2}$

(h) $\lim_{x\to 0^+} \dfrac{1}{x}$

(i) $\lim_{x\to 0^+} (1-2x)^{\frac{1}{x}}$ - something to do with e or log

(j) $\lim_{x\to 0^+} \left(1-2x^2\right)^{\frac{1}{x}}$

(k) $\lim_{x\to\infty} \left(1-\dfrac{2}{e^x}\right)^x$

(l) $\lim_{x\to\infty} \left(1-\dfrac{2}{x}\right)^{e^x}$

(m) $\lim_{n\to\infty} \dfrac{1}{n!}$ - maybe useful when we do series...

(n) $\lim_{n\to\infty} \dfrac{n^3}{n!}$

(o) $\lim_{n\to\infty} \dfrac{n^3}{n!}\dfrac{(n+1)!}{(n+1)^3}$

(p) $\lim_{n\to\infty} \dfrac{n!}{n^3}\dfrac{(n+1)^3}{(n+1)!}$

2. Do the series converge or diverge? Justify.

(a) $\sum_{k=1}^{\infty} \dfrac{1}{2k} - \dfrac{1}{2k+2}$ - Telescoping

(b) $\sum_{k=1}^{\infty} \dfrac{1}{\sqrt{k}} - \dfrac{1}{\sqrt{k+1}}$

(c) $\displaystyle\sum_{k=1}^{\infty} \sqrt{k} - \sqrt{k+1}$

(d) $\displaystyle\sum_{k=1}^{\infty} \frac{2}{3^k}$ - Geometric Series

(e) $\displaystyle\sum_{k=-3}^{\infty} \frac{1}{7^k}$

(f) $\displaystyle\sum_{k=1}^{\infty} 3^k$

(g) $\displaystyle\sum_{k=1}^{\infty} \frac{2^k}{2^k+1}$ - Divergence Test

(h) $\displaystyle\sum_{k=1}^{\infty} \left(1 - \frac{1}{k}\right)^k$

(i) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2+1}$ - Integral test

(j) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k\ln(k)}$

(k) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k}$ - P-Series

(l) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{\sqrt{k}}$

(m) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2}$

(n) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k!}$ - ratio test

(o) $\displaystyle\sum_{k=1}^{\infty} \frac{2^k}{k!}$

(p) $\displaystyle\sum_{k=1}^{\infty} \frac{k!}{2^k}$

(q) $\displaystyle\sum_{k=1}^{\infty} \frac{k^k}{(2k)!}$

(r) $\displaystyle\sum_{k=1}^{\infty}\left(1-\frac{1}{k}\right)^{k^2}$ - root test

(s) $\displaystyle\sum_{k=1}^{\infty}\left(1+\frac{1}{k}\right)^{k^2}$

(t) $\displaystyle\sum_{k=1}^{\infty}\left(\frac{2k^2+1}{3k^2+2k+1}\right)^{k}$

(u) $\displaystyle\sum_{k=1}^{\infty}\left(\frac{3k^2+2k+1}{2k^2+1}\right)^{k}$