

Math 2320 - Series Worksheet

1 Telescoping

1. $\sum_{k=1}^{\infty} \frac{1}{k} - \frac{1}{k+2}$

2. $\sum_{k=1}^{\infty} \sqrt{k+1} - \sqrt{k}$

3. $\sum_{k=1}^{\infty} \frac{1}{k(k+1)}$

2 Geometric Series

1. $\sum_{k=1}^{\infty} 3^{-k}$

2. $\sum_{k=1}^{\infty} \frac{1}{3^{-k}}$

3. $\sum_{k=17}^{\infty} 11 \cdot 3^{-k}$

4. $\sum_{k=1}^{\infty} \frac{2^k + 3^k}{4^k}$

5. $\sum_{k=1}^{\infty} \frac{4^k}{2^k + 3^k}$

6. $9, -3, 1, -\frac{1}{3}, -\frac{1}{9}, \dots$

3 Integral Test

1. $\sum_{k=1}^{\infty} e^{-k}$

$$2. \sum_{k=1}^{\infty} \frac{1}{1+k^2}$$

$$3. \sum_{k=17}^{\infty} \frac{1}{k \ln(k)}$$

$$4. \sum_{k=17}^{\infty} \frac{1}{k \ln^2(k)}$$

4 P-Series Test

$$1. \sum_{k=1}^{\infty} \frac{1}{\sqrt[4]{k}}$$

$$2. \sum_{k=1}^{\infty} \frac{1}{k^2}$$

$$3. \sum_{k=1}^{\infty} k^3$$

$$4. \sum_{k=1}^{\infty} k^{-3}$$

$$5. \sum_{k=1}^{\infty} \frac{k+k^2}{k^3}$$

$$6. \sum_{k=1}^{\infty} \frac{k+k^2}{k^4}$$

5 Ratio Test

$$1. \sum_{k=1}^{\infty} \frac{k}{2^k}$$

$$2. \sum_{k=1}^{\infty} \frac{1}{k!}$$

$$3. \sum_{k=1}^{\infty} \frac{2^k}{k!}$$

$$4. \sum_{k=1}^{\infty} \frac{k!}{k^k}$$

$$5. \sum_{k=1}^{\infty} \frac{(k!)^2}{k^k}$$

$$6. \sum_{k=1}^{\infty} \frac{(k!)^2}{(2k)!}$$

$$7. \sum_{k=1}^{\infty} \frac{(2k)!}{k^k}$$

6 Root Test

$$1. \sum_{k=1}^{\infty} \left(\frac{k^2 + 1}{2k^2 - 1} \right)^k$$

$$2. \sum_{k=1}^{\infty} \left(\frac{3k^2 + 1}{2k^2 - 1} \right)^k$$

$$3. \sum_{k=1}^{\infty} \left(1 - \frac{1}{k} \right)^{k^2}$$

$$4. \sum_{k=1}^{\infty} \left(1 + \frac{2}{k} \right)^{k^2}$$

7 Comparison Test

$$1. \sum_{k=1}^{\infty} \frac{1}{k^3 + 1}$$

$$2. \sum_{k=1}^{\infty} \frac{1}{k^2 + 1}$$

$$3. \sum_{k=3}^{\infty} \frac{1}{k-1}$$

$$4. \sum_{k=2}^{\infty} \frac{1}{\sqrt{k}-1}$$

8 Limit Comparison Test

$$1. \sum_{k=1}^{\infty} \frac{3k^2+1}{2k^5-1}$$

$$2. \sum_{k=1}^{\infty} \sqrt{\frac{3k^2+1}{2k^3-1}}$$

$$3. \sum_{k=1}^{\infty} \sqrt{\frac{3k^2+1}{2k^4-1}}$$

$$4. \sum_{k=1}^{\infty} \sqrt{\frac{3k^2+1}{2k^5-1}}$$

$$5. \sum_{k=1}^{\infty} \frac{\sin(\frac{1}{k})}{\sqrt{k}}. \text{ Hint: try comparing to } \sum b_k = \sum \frac{1}{k^{3/2}}.$$

9 Alternating Series Test

$$1. \sum_{k=1}^{\infty} \frac{1}{k} (-1)^k$$

$$2. \sum_{k=1}^{\infty} \frac{k}{k+1} (-1)^k$$

$$3. \sum_{k=1}^{\infty} \left(1 - \frac{1}{k}\right)^k (-1)^k$$