2320 §I

① $\int x^2\sqrt{3-4x^3}\,dx$ $\quad u = 3-4x^3$

$du = -12x^2dx$

$= -\frac{1}{12}\int (u)^{1/2}\,du$ $\quad -\frac{1}{12}du = x^2dx$

$= -\frac{1}{12}\,\frac{u^{3/2}}{3/2} + C = \boxed{-\frac{1}{12}\,\frac{(3-4x^3)^{3/2}}{3/2} + C}$

② $\int e^{3x}\,dx$ $\quad u = 3x$

$du = 3dx$

$= \int e^{u}\cdot\frac{1}{3}du$ $\quad \frac{1}{3}du = dx$

$= \frac{1}{3}e^{u} + C = \boxed{\frac{1}{3}e^{3x} + C}$

~~③ $\int \frac{x^2}{4-3x^2}dx$ $\quad u = 4-3x^2$ $\quad du = -12x\,dx$ $\quad -\frac{1}{12}du =$~~

③ $\int \frac{x^2}{3-4x^3}dx$ $\quad u = 3-4x^3$

$du = -12x^2dx$

$= \int \frac{-\frac{1}{12}du}{u}$ $\quad -\frac{1}{12}du = x^2dx$

$= -\frac{1}{12}\ln|u| + C$

$= \boxed{-\frac{1}{12}\ln|3-4x^3| + C}$

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(4.) $\int \frac{x^2}{1+(3-4x^3)^2}dx$

$u = 3-4x^3$

$du = -12x^2\,dx$

$-\frac{1}{12}du = x^2dx$

$= \int \frac{-\frac{1}{12}}{1+u^2}\,du$

$= -\frac{1}{12}\tan^{-1}(u) + C = -\frac{1}{12}\tan^{-1}(3-4x^3)+C$

(5.) $\int \frac{x\sin(x^2)}{\cos(x^2)}dx$

$u = \cos(x^2)$

$du - 2x\sin(x^2)\,dx$

$-\frac{1}{2}du = x\sin(x^2)dx$

$= \int \frac{-1/2}{u}\,du$

$= -1/2 \ln|u| + C$

$= -1/2 \ln|\cos(x^2)| + C$

(6.) $\int x^2 e^{3x}dx$

$u = x^2$ $\quad dv = e^{3x}dx$

$du = 2x\,dx$ $\quad v = \frac{1}{3}e^{3x}$

$= x^2\cdot\frac{1}{3}e^{3x} - \int 2x\cdot\frac{1}{3}e^{3x}dx$

$= \frac{x^2e^{3x}}{3} - \frac{2}{3}\int x\,e^{3x}dx$

$u = x$ $\quad dv = e^{3x}dx$

$du = dx$ $\quad v = \frac{1}{3}e^{3x}$

$= \frac{x^2e^{3x}}{3} - \frac{2}{3}\left[x\cdot\frac{1}{3}e^{3x} - \int\frac{1}{3}e^{3x}dx\right]$

$= \frac{x^2e^{3x}}{3} - \frac{2}{3}\left[\frac{1}{3}xe^{3x} - \frac{1}{9}e^{9x}\right] + C$

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(7) $\int x \sin(4x)\,dx$

$u = x$ $\quad dv = \sin(4x)\,dx$

$du = dx$ $\quad v = -\frac{1}{4}\cos(4x)$

$= x \cdot \left(-\frac{1}{4}\cos(4x)\right) - \int \left(-\frac{1}{4}\right)\cos(4x)\,dx$

$= \boxed{-\frac{1}{4} x \cos(4x) + \frac{1}{16}\sin(4x) + C}$

(8) $\int \ln(x)\,dx$

$u = \ln(x)$ $\quad dv = dx$

$du = \frac{1}{x}\,dx$ $\quad V = x$

$= x \cdot \ln(x) - \int x \cdot \frac{1}{x}\,dx$

$= x \cdot \ln(x) - \int (1)\,dx = \boxed{x\ln(x) - x + C}$

(9) $\int x^2 \ln(x)\,dx$

$u = \ln(x)$ $\quad dv = x^2\,dx$

$du = \frac{1}{x}\,dx$ $\quad V = \frac{1}{3}x^3$

$= \frac{1}{3}x^3\ln(x) - \int \frac{1}{3}x^3 \cdot \frac{1}{x}\,dx$

$= \frac{1}{3}x^3\ln(x) - \frac{1}{3}\int x^2\,dx$

$= \frac{1}{3}x^3\ln(x) - \frac{1}{3} \cdot \frac{1}{3}x^3 + C$

$= \frac{1}{3}x^3\ln(x) - \frac{1}{9}x^3 + C$

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(10.) $\int \sec^3(x)\,dx$ $\quad u = \sec(x) \quad dv = \sec^2(x)\,dx$

$du = \sec(x)\tan(x)\,dx \quad v = \tan(x)$

$= \sec(x)\tan(x) - \int \tan(x) \cdot \sec(x)\tan(x)\,dx$

$= \sec(x)\tan(x) - \int \tan^2(x)\sec(x)\,dx$

$-\int (\sec^2(x) - 1)\sec(x)\,dx$

$-\int \sec^3(x)\,dx + \int \sec(x)\,dx$

So $\quad -\int \sec^3(x)\,dx + \ln|\sec(x) + \tan(x)| + C$

$\underline{\int \sec^3(x)\,dx} = \sec(x)\tan(x) + \ln|\sec(x) + \tan(x)| - \underline{\int \sec^3(x)\,dx}$

$+\int \sec^3(x)\,dx \qquad\qquad +\int \sec^3(x)\,dx$

$$\frac{\cancel{2}\int \sec^3(x)\,dx}{\cancel{2}} = \frac{\sec(x)\tan(x) + \ln|\sec(x) + \tan(x)| + C}{2}$$

(11.) $\int \sin^3(x)\cos(x)\,dx \quad u = \sin(x) \quad du = \cos(x)\,dx$

$= \int u^3\,du$

$= \frac{1}{4}u^4 + C = \frac{1}{4}\sin^4(x) + C$

(12.) $\int \sin^3(x)\cos^2(x)\,dx$

$$= \int \sin^2(x)\cos^2(x)\cdot \sin(x)\,dx \qquad u=\cos(x)$$
$$du = -\sin(x)\,dx$$

$$= \int (1-\cos^2(x))\cos^2(x)\,\sin(x)\,dx$$

$$= -\int (1-u^2)\,u^2\,du$$

$$= -\int u^2 - u^4\,du = -\frac{u^3}{3} + \frac{u^5}{5} = -\frac{\cos^3(x)}{3} + \frac{\cos^5(x)}{5} + C$$

(13.) $\int \sin^{1/2}(x)\cos^3(x)\,dx$

$$= \int \sin^{1/2}(x)\cos^2(x)\,\cos(x)\,dx \qquad u=\sin(x)$$
$$= \int \sin^{1/2}(x)(1-\sin^2(x))\cos(x)\,dx \qquad du=\cos(x)\,dx$$

$$= \int u^{1/2}\cdot(1-u^2)\,du = \int u^{1/2} - u^{5/2}\,du = \frac{u^{3/2}}{3/2} - \frac{u^{7/2}}{7/2} + C$$

$$= \frac{2}{3}(\sin(x))^{3/2} - \frac{2}{7}(\sin(x))^{3/2} + C$$

(14.) $\int \sin^2(3x)\cos^2(3x)\,dx$

$$= \int \tfrac{1}{2}[1-\cos(6x)]\cdot\tfrac{1}{2}[1+\cos(6x)]\,dx$$

$$= \frac{1}{4}\int 1 - \cos^2(6x)\,dx = \frac{1}{4}\int 1 - \frac{1}{2}(1+\cos(12x))\,dx$$

$$= \frac{1}{4}\int \frac{1}{2} - \frac{1}{2}\cos(12x)\,dx = \frac{1}{4}\left[\frac{1}{2}x - \frac{1}{2}\cdot\frac{1}{12}\cdot\sin(12x)\right] + C$$

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(15.) $\int \sin^4(2x)\,dx = \int \left(\sin^2(2x)\right)^2 dx$

$= \int \left(\frac{1}{2}\left(1 - \cos(4x)\right)\right)^2 dx$ $\qquad \sin^2(2x) = \frac{1}{2}\left(1 - \cos(4x)\right)$

$= \frac{1}{4}\int 1 - 2\cos(4x) + \cos^2(4x)\,dx$ $\qquad \cos^2(4x) =$

$= \frac{1}{4}\left[x - 2 \cdot \frac{1}{4}\sin(4x)\right] + \frac{1}{4}\int \frac{1}{2}\left(1 + \cos(8x)\right)dx$

$= \frac{1}{4}\left[x - \frac{1}{2}\sin(4x)\right] + \frac{1}{8}\left(x + \frac{1}{8}\sin(8x)\right) + C$

(16.) $\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}(x) + C$

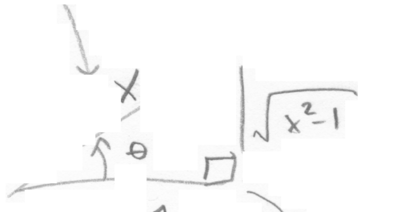
OR you could use $x = \sin\theta$ as a sub.

(17.) $\int \frac{1}{\sqrt{x^2-1}}\,dx$

$x = \sec(\theta)$

$dx = \sec\theta \tan\theta \, d\theta$

$= \int \frac{\sec(\theta)\tan(\theta)}{\sqrt{\sec^2\theta - 1}}\,d\theta$



$\tan\theta = \frac{\sqrt{x^2-1}}{1}$

$= \int \frac{\sec(\theta)\tan\theta}{\sqrt{\tan^2\theta}}\,d\theta = \int \frac{\sec(\theta)\cancel{\tan\theta}}{\cancel{\tan\theta}}\,d\theta$

$= \ln|\sec\theta + \tan\theta| + C = \ln\left|x + \sqrt{x^2-1}\right| + C$

(18.) $\int \sqrt{1+x^2}\, dx$

$x = \tan\theta$

$dx = \sec^2\theta \, d\theta$

$\frac{x}{1} = \tan\theta$

$\sec\theta = \frac{H}{A} = \sqrt{}$

$= \int \sqrt{1+\tan^2(\theta)}\, \sec^2\theta \, d\theta$

$= \int \sqrt{\sec^2\theta}\, \sec^2\theta \, d\theta = \int \sec^3\theta \, d\theta$

FROM (10)

$= \frac{1}{2}\sec(\theta)\tan\theta + \frac{1}{2}\ln|\sec\theta + \tan\theta| + C$

$= \boxed{\frac{1}{2}(\sqrt{1+x^2})\cdot x + \frac{1}{2}\ln|\sqrt{1+x^2} + x| + C}$

(19.) $\int \frac{1}{\sqrt{1+x^2}}\, dx$

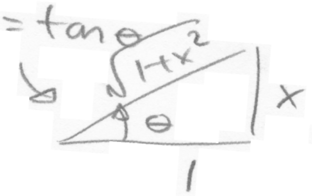
$x = \tan\theta$

$dx = \sec^2(\theta)\, d\theta$

$= \int \frac{\sec^2\theta \, d\theta}{\sqrt{1+\tan^2\theta}} = \int \frac{\sec^2(\theta)}{\sqrt{\sec^2\theta}}\, d\theta = \int \sec(\theta)\, d\theta$

$= \ln|\sec\theta + \tan\theta| + C = \boxed{\ln|\sqrt{1+x^2} + x| + C}$

$\frac{x}{1} = \tan\theta$



$\longrightarrow \sec\theta = \frac{H}{A} = \frac{\sqrt{1+x^2}}{1}$

§ I

(20.) $\int \frac{2-x^2}{x^2(1+x^2)}\,dx$ $\qquad \frac{2-x^2}{x^2(1+x^2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+1}$

$$2 - x^2 = Ax(x^2+1) + B(x^2+1) + (Cx+D)x^2$$

$$2 - x^2 = Ax^3 + Ax + Bx^2 + B + Cx^3 + Dx^2$$

$$2 - x^2 = x^3(A+C) + x^2(B+D) + Ax + B$$

x^3: $0 = A + C$ $\rightarrow$ $0 = 0 + C$

$\boxed{0 = C}$

x^2: $-1 = B + D$

x^1: $\boxed{0 = A}$ $\rightarrow$ $-1 = 2 + D$

$\boxed{-3 = D}$

1: $\boxed{2 = B}$

So $\frac{2-x^2}{x^2(1+x^2)} = \frac{0}{x} + \frac{2}{x^2} + \frac{0x-3}{1+x^2} = \frac{2}{x^2} - \frac{3}{1+x^2}$

So $\int \frac{2-x^2}{x^2(1+x^2)}\,dx = \int \frac{2}{x^2} - \frac{3}{1+x^2}\,dx$

$$= \boxed{-\frac{2}{x} - 3\tan^{-1}(x) + C}$$

§ I

(21) $\int \frac{6x^2-4}{x(x^2-1)}\,dx$ | $\frac{6x^2-4}{x(x^2-1)} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x-1}$

$6x^2-4 = A(x+1)(x-1) + Bx(x-1) + Cx(x+1)$

$= Ax^2 - A + Bx^2 - Bx + Cx^2 + Cx$

$6x^2-4 = (A+B+C)x^2 + (-B+C)x - A$

$\underline{\underline{x^2}}:\ 6 = A+B+C$

$\underline{\underline{x^1}}:\ 0 = -B+C$

$\underline{1}:\ -4 = -A \Rightarrow \boxed{A=4}$

$6 = 4+B+C$
$0 = -B+C$ +
$6 = 4 + 2C$
$2 = 2C$
$\boxed{1=C}$
$\boxed{B=1}$

So

$\int \frac{6x^2-4}{x(x^2-1)}\,dx = \int \frac{4}{x} + \frac{1}{x+1} + \frac{1}{x-1}\,dx$

$= \boxed{4\ln|x| + \ln|x+1| + \ln|x-1| + C}$

(22.) $\displaystyle\int \frac{3x^3+x+1}{x^3-x}\,dx$

FIRST NOTE FRACTION IS IMPROPER $\longrightarrow$

$$
\begin{array}{r l}
 & 3 \quad R\ 4x+1 \\
x^3-x \,\big)\!\! & 3x^3+0x^2+x+1 \\
- & (3x^3 \quad - 3x) \\
 & \qquad\qquad 4x+1
\end{array}
$$

So

$$\frac{3x^3+x+1}{x^3-x} = 3 + \frac{4x+1}{x^3-x} = 3 + \frac{4x+1}{x(x+1)(x-1)}$$

Let

$$\frac{4x+1}{x(x+1)(x-1)} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x-1}$$

work SIMILAR TO #21

$$= \frac{-1}{x} + \frac{-3/2}{x+1} + \frac{5/2}{x-1}$$

So

$$\int \frac{x^3+x+1}{x^3-x}\,dx = \int 3 + \frac{-1}{x} + \frac{-3/2}{x+1} + \frac{5/2}{x-1}\,dx$$

$$= \boxed{3x - \ln|x| + \tfrac{-3}{2}\ln|x+1| + \tfrac{5}{2}\ln|x-1| + C}$$

§ 3

1. (a) $\lim_{n\to\infty} \dfrac{n^3-1}{1-7n^3} = \dfrac{1}{-7}$

(b.) $\lim_{n\to\infty} \dfrac{n^2-1}{1-7n^3} = 0$

(c.) $\lim_{n\to\infty} \dfrac{n^3-1}{1-7n^2} = -\infty$

(d) $\lim_{x\to 0} \dfrac{\sin(x)}{x} \overset{(LH)}{=} \lim_{x\to 0} \dfrac{\cos(x)}{1} = \cos(0) = 1$

(e) $\lim_{x\to 0} \dfrac{1-e^{x^2}}{e^{x^2}} = \dfrac{1-e^0}{e^0} = \dfrac{0}{1} = \boxed{0}$

(f) $\lim_{x\to 0} \dfrac{1}{x}$ DNE

(g) $\lim_{x\to 0} \dfrac{1}{x^2} = +\infty$

(h) $\lim_{x\to 0^+} \dfrac{1}{x} = +\infty$

(i) $\lim (1-2x)^{1/x} = e^{-2}$

OR

$L = \lim_{x\to 0^+} (1-2x)^{1/x}$

$\ln L = \lim \ln (1-2x)^{1/x}$

$\ln L = \lim \frac{1}{x} \ln(1-2x) = \lim \dfrac{\ln(1-2x)}{x} \overset{(LH)}{=} \lim_{x\to 0^+} \dfrac{\frac{1}{1-2x}(-2)}{1} = -2$

so $\ln L = -2$

$\boxed{L = e^{-2}}$

(j) $L = \lim_{x\to 0} (1-2x^2)^{1/x}$

$\ln L = \lim_{x\to 0} \frac{\ln(1-2x^2)}{x} \overset{LH}{=} \lim_{x\to 0^+} \frac{\frac{1}{1-2x^2}(-2x)}{1} = 0$

so $L = e^0 = 1$

(k) $L = \lim_{x\to\infty} \left(1-\frac{2}{e^x}\right)^x$

$\ln L = \lim_{x\to\infty} x \ln\left(1-\frac{2}{e^x}\right) = \lim_{x\to\infty} \frac{\ln(1-2e^{-x})}{x^{-1}}$

$\overset{LH}{=} \lim_{x\to\infty} \frac{\frac{1}{1-2e^{-x}}(2e^{-x})}{-x^{-2}} = \lim_{x\to\infty}\left(\frac{2e^{-x}}{1-2e^{-x}} \cdot \frac{x^2}{1}\right)$

$= \lim_{x\to\infty} \frac{2x^2}{e^x-2} = 0$

so $\ln L = 0$

$\Rightarrow L = e^0 = 1$

(l) $L = \lim_{x\to\infty} \left(1-\frac{2}{x}\right)^{e^x}$

$\ln L = \lim_{x\to\infty} e^x \cdot \ln\left(1-\frac{2}{x}\right) = \lim_{x\to\infty} \frac{\ln(1-2x^{-1})}{e^{-x}}$

$\overset{LH}{=} \lim_{x\to\infty} \frac{\frac{1}{1-2x^{-1}}(0+2x^{-2})}{-e^{-x}} = \lim_{x\to\infty} \frac{2x^{-2}}{1-2x^{-1}} \cdot \left(\frac{-e^x}{1}\right) = \infty$

$\ln L = \infty$

so $\boxed{L = \infty}$

2320

(m) $\lim \frac{1}{n!} = 0$

(n) $\lim_{n\to\infty} \frac{n^3}{n!} = 0$ (why?)

(o) $\lim_{n\to\infty} \frac{n^3}{n!} \cdot \frac{(n+1)!}{(n+1)^3} = \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^3 \cdot \frac{n+1}{1} = \infty$

(p) $\lim \frac{n!}{n^3} \cdot \frac{(n+1)^3}{(n+1)!} = \qquad = 0$

(2) $2 \sum \frac{1}{2k} - \frac{1}{2k+2}$

I believe I did just about all of these in the Review.