MA 2310 Practice Test 1

There are some answers below which maybe helpful. Notice I did not include all work. For example 2: 2a and 3:1a have no work shown but you must display all work for credit. Some problems require little or no work for example in 2: 1f, 1g, 1h and in 3:2a.

1 Preliminaries

1. Know the trigonometry we discussed without a calculator (as on the quizzes).

2. Know how to graph basic functions such as: $y = 3x - 1$, $y = 2x^2$, $y = x^3$, $y = \sqrt{x}$, $y = \sin(x)$, $y = \frac{1}{x}$, $y = \frac{1}{x^2}$, $y = e^x$, $y = \ln(x)$, $y = x^{0.57}$ $x^2 + y^2 = 4$ and $y = \cos(x)$.

3. Know how to identify a functions domain.

2 Limits

1. Compute the following limits and show your work.

   (a) $\lim_{x \to 3^-} \frac{x-3}{|x-3|} = -1$

   (b) $\lim_{x \to 2} \frac{|x-2|}{(x-2)^2} = +\infty$

   (c) $\lim_{x \to 2} \frac{x^2-8}{|x-2|} = \text{DNE}$

   (d) $\lim_{x \to 2} \frac{1}{|x-2|} = +\infty$

   (e) $\lim_{x \to 2} \frac{1}{x-2} = \text{DNE}$

   (f) $\lim_{x \to \infty} \frac{9x^3-1}{7x^5+5} = 0$

   (g) $\lim_{x \to \infty} \frac{9x^5-1}{7x^3+5} = +\infty$

   (h) $\lim_{x \to \infty} \frac{9x^5-1}{7x^5+5} = 9/7$

   (i) $\lim_{x \to \infty} e^{-x} = 0$

   (j) $\lim_{x \to \infty} \ln(x) = +\infty$

   (k) $\lim_{x \to \infty} \frac{7}{\ln(x)} = 0$

   (l) $\lim_{x \to -\infty} \frac{9x^3-1}{7x^5+5} = 0$

   (m) $\lim_{x \to -\infty} \frac{9x^5-1}{7x^3+5} = +\infty$

(n) $\lim_{x\to-\infty} \frac{9x^5-1}{7x^5+5} = 9/7$

(o) $\lim_{x\to-\infty} e^{-x} = +\infty$

(p) $\lim_{x\to-\infty} \ln(x)$ DNE

(q) $\lim_{x\to 0} \frac{\sin(2x^2)}{x^2} = \lim_{x\to 0} \frac{2\sin(2x^2)}{2x^2} = 2$

(r) $\lim_{x\to 0} \frac{\cos(3x)}{x^2} = +\infty$

(s) $\lim_{x\to 0} \frac{\tan(3x)}{3x} = \lim_{x\to 0} \frac{\sin(3x)}{\cos(3x)3x} = \lim_{x\to 0} \frac{\sin(3x)}{3x} \frac{1}{\cos(3x)} = 1$

2. For the following functions compute the horizontal and vertical aymptotes.

(a) $f(x) = \ln(x)$
HA: $y = 0$ VA: $x = 0$

(b) $f(x) = \frac{1}{x}$
HA: $y = 0$ VA: $x = 0$

(c) $f(x) = \frac{x^2-1}{x+1}$
HA: none VA: none

(d) $f(x) = \frac{x^2+1}{x+1}$
HA: none VA: $x = -1$

(e) $f(x) = \frac{x^2-1}{x^3+1}$ Hint $x^3 + 1 = (x + 1)(\ldots$
HA: $y = 0$ VA:none

(f) $f(x) = \frac{\sin(x)}{x}$
HA: $y = 0$ VA: none

3. Determine if the function can be made continuous at the point x_0. And if it can find the value c so that the function is continuous at x_0.

(a) $f(x) = \begin{cases} \frac{\sin(x)}{x} & : x \neq 0 \\ c & : x = 0 \end{cases}$ where $x_0 = 0$.

Answer: Since $\lim_{x\to 0} \frac{\sin(x)}{x} = 1$, setting $c = 1$ makes f continuous at $x = 0$.

(b) $f(x) = \begin{cases} \frac{x^2-1}{x+1} & : x \neq -1 \\ c & : x = -1 \end{cases}$ where $x_0 = -1$.

Answer: Since $\lim_{x\to -1} \frac{x^2-1}{x+1} = -2$, setting $c = -2$ makes f continuous at $x = -1$.

(c) $f(x) = \begin{cases} \frac{x^2-4}{x+1} & : x \neq -1 \\ c & : x = -1 \end{cases}$ where $x_0 = -1$.

Answer: Since $\lim_{x\to -1} \frac{x^2-4}{x+1}$ DNE, there is no numeric limit at $x = -1$ so f cannot be made continous at $x = -1$. Note that at $x = -1$ f has a vertical asymptote.

3 Derivatives

1. Compute the following derivatives from the definition (you must use the definition).

   (a) $f(x) = x^2$ $\quad f'(x) = 2x$

   (b) $f(x) = 3x - 2$ $\quad f'(x) = 3$

   (c) $f(x) = \frac{1}{x^2}$ $\quad f'(x) = -2x^{-3}$

   (d) $f(x) = \sqrt{x}$ $\quad f'(x) = \frac{1}{2\sqrt{x}}$

   (e) $f(x) = \sin(x)$ $\quad f'(x) = \cos(x)$

   (f) $f(x) = \begin{cases} \frac{x^2}{|x|} & : x \neq 0 \\ 0 & : x = 0 \end{cases}$ For this one only compute the derivative at the point $x = 0$.
   $f'(0) = 0$

2. Compute the following derivatives from the rules.

   (a) $f(x) = x^2 - 1 - \frac{1}{x}$ $\quad f'(x) = 2x + x^{-2}$

   (b) $f(x) = 3x - 2$ $\quad f'(x) = 2$

   (c) $f(x) = x(3x^2 - 2)$ $\quad f'(x) = 9x^2 - 2$

   (d) $f(x) = \frac{\sqrt{x}+1}{x}$ $\quad f'(x) = \frac{-1}{2}x^{-3/2} - x^{-2}$

   (e) $f(x) = \sin(x)$ $\quad f'(x) = \cos(x)$

3. Find the equation of the tangent line at the indicated point $x = a$.

   (a) $f(x) = -3x^2 + 2$; $a = 1$ $\quad y - (-1) = -6(x - 1)$
   The work: We need two things for the equation of the line: a point and a slope. The x-coordinate of the point is given to us $a = 1$. We find the y-coordinate by computing $f(1) = -3(1)^2 + 2 = -1$. So now we have our point $(1, -1)$.

   Next we need to find the slope. First compute the derivative $f'(x) = -6x$. Then recall that the slope of the tangent line at $a = 1$ is $m = f(1) = -6$.

   Now equipped with a point $(1, -1)$ and a slope $m = -6$, we use the point-slope formula for a line

   $$y - y_0 = m(x - x_0)$$

   $$y - (-1) = -6(x - 1)$$

   $$y = -6x + 5$$

   (b) $f(x) = e^x$; $a = 0$ $\quad$ Do not do this one

   (c) $f(x) = e^x$; $a = 1$ $\quad$ Do not do this one

   (d) $f(x) = \sin(x)$; $a = \pi/4$ $\quad y - \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}(x - \pi/4)$