Math 2310 - Practice Final Exam

1 Derivatives

1. Write the definition of the derivative.
2. Use the definition of the derivative to compute the derivative of $f(x) = x^2$.
3. Use the definition of the derivative to compute the derivative of $f(x) = x^3$.
4. Use the definition of the derivative to compute the derivative of $f(x) = \sqrt{x}$. - recall we multiplied the limit by $\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$.

2 Related Rates

1. The volume of a spherical ballon is increasing at the rate of 100 cubic inches per second. When the radius is 7 inches, how fast is the radius increasing?
2. The radius of a spherical ballon is increasing at the rate of 0.5 inches per second. When the radius is 4 inches, how fast is the volume increasing?
3. The boat is racing down the center of of a river at 3 meters per second. We are standing on the side of the river watching the boat go down river (the center of the river is 20 meters from us). How fast is the boat moving away from us when the boat is 12 meters downriver of us?
4. My chemtristry major friend gave me the following formula
$$PV = nRT.$$
She said that R and n are constants. If we know that $\frac{dV}{dt} = 2$ and $\frac{dT}{dt} = -2$ when $V = 10$ and $P = 7$, find $\frac{dP}{dt}$ in terms of n and R.
5. My physics major friend gave me the following formula
$$F = G\frac{m_1 M_2}{R^2}.$$
She said that G, m_1 and m_2 are constants. If we know that $\frac{dR}{dt} = 2$ when $R = 10$, find $\frac{dF}{dt}$ in terms of G, m_1 and m_2.

3 Graphing, increasing decreasing and max min problems

1. For the function $f(x) = (x+1)^2(x-1)^3$ find the first derivative number line, the critical points and classify the critical points as maximum or minimum.

2. For the function $f(x) = e^{x^3-x}$ find the first derivative number line, the critical points and classify the critical points as maximum or minimum.

3. For the function $f(x) = x^4-4x^3+4x^2+3$ find the first derivative number line, the critical points and classify the critical points as maximum or minimum.

4. For the function $f(x) = \frac{x^2+3}{x-1}$ find the first derivative number line, the critical points and classify the critical points as maximum or minimum.

5. For the function $f(x) = x^3 - 12x^2 + 3$ find the first derivative number line, the second derivative number line, the critical points and classify the critical points as maximum or minimum. And find the Possible points of inflection and classify. And graph labeling each of the critical points.

6. For the function $f(x) = x^4 - 12x^3 + 30x^2 + 3$ find the first derivative number line, the second derivative number line, the critical points and classify the critical points as maximum or minimum. And find the Possible points of inflection and classify. And graph labeling each of the critical points.

4 Optimization Problems

1. The various pen problems.

2. The various can cost problem.

5 Limits

1. $\lim_{n\to\infty} \dfrac{n^2+3n-1}{2n^2+1}$

2. $\lim_{n\to\infty} \dfrac{n^3+3n-1}{2n^2+1}$

3. $\lim_{n\to\infty} \dfrac{n^2+3n-1}{2n^3+1}$

4. $\lim_{x\to 2} \dfrac{x^2-4}{x-2}$ - do this problem without L'Hopital's Rule

5. $\lim_{x\to 0} \dfrac{1}{x}$

6. $\lim_{x\to 0} \dfrac{1}{x^2}$

7. $\lim_{x\to 0^+} \dfrac{1}{x}$

8. $\lim_{x\to 0^+} \dfrac{1}{x}$

5.1 L'Hopital's Rule ($\frac{0}{0}$ or $\frac{\infty}{\infty}$)

1. $\lim_{n\to\infty} \dfrac{n^2}{e^n}$

2. $\lim_{n\to\infty} \dfrac{n^2}{\ln(n)}$

3. $\lim_{n\to\infty} \dfrac{\ln(n)}{n^2+1}$

4. $\lim_{x\to 0} \dfrac{xe^x}{1-e^x}$

5. $\lim_{x\to 0} \dfrac{xe^x}{(1-e^x)^2}$

6. $\lim_{x\to 0} \dfrac{1-e^{x^2}}{1-e^x}$

7. $\lim_{x\to 0} \dfrac{1-e^x}{1-\cos(x)}$

8. $\lim_{x\to 0} \dfrac{1-\cos(x)}{1-e^x}$

9. $\lim_{x\to 0} \dfrac{1-e^{x^2}}{1-\cos(x)}$

5.2 L'Hopital's Rule (0^0, $0 \cdot \infty$, 1^∞ or ∞^0

1. $\lim_{x \to 0} \frac{1}{x} \tan(x)$
2. $\lim_{x \to 0} \frac{1}{x} \tan^{-1}(x)$
3. $\lim_{x \to 0^+} (1-x)^{\frac{1}{x}}$
4. $\lim_{x \to 0} (1+x)^{\frac{1}{x^2}}$
5. $\lim_{x \to 0^+} \left(1+x^2\right)^{\frac{1}{x^2}}$
6. $\lim_{n \to \infty} (1+\frac{3}{n})^7$
7. $\lim_{n \to \infty} (1+\frac{3}{n})^n$
8. $\lim_{n \to \infty} (1+\frac{3}{n^2})^n$
9. $\lim_{n \to \infty} (1+\frac{3}{n^2})^{n^2}$
10. $\lim_{x \to 0^+} (\sin(x))^x$
11. $\lim_{x \to 0^+} x^{\sin(x)}$
12. $\lim_{n \to \infty} n^{\sin(\frac{1}{n})}$
13. $\lim_{n \to \infty} [n^2+1]^{\frac{1}{\ln(n)}}$

6 Antiderivatives

1. $\int \frac{1}{x}\, dx$
2. $\int x^2 - \frac{1}{x} + e^x\, dx$
3. $\int \frac{1}{x^2+1}\, dx$

4. $\int e^{3x+1}\,dx$ - Use u-sub, let $u = 3x + 1$.

5. $\int xe^{3x^2+1}\,dx$ - Use u-sub, let $u = 3x^2 + 1$.

6. $\int \frac{x}{x^2+1}\,dx$

7. $\int x\sqrt{x^2+1}\,dx$

8. $\int x\sin(x^2+1)\,dx$

9. $\int x\sec^2(x^2+1)\,dx$