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§ I

① $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

② $f'(x) = \lim_{h\to 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h\to 0} \frac{x^2+2xh+h^2 - h^2}{h}$

$= \lim_{h\to 0} \frac{2xh+h^2}{h} = \lim_{h\to 0} 2x+h = \boxed{2x}$

③ $f'(x) = \lim_{h\to 0} \frac{(x+h)^3 - x^3}{h} = \lim_{h\to 0} \frac{x^3+3x^2h+3xh^2+h^3-x^3}{h}$

$= \lim_{h\to 0} 3x^2 + 3xh + h^2 = 3x^2$

④ $F'(x) = \lim_{h\to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$

$= \lim \frac{\cancel{x}+h - \cancel{x}}{\cancel{h}(\sqrt{x+h}+\sqrt{x})} = \lim_{h\to 0} \frac{1}{\sqrt{x+h}+\sqrt{x}} = \frac{1}{\sqrt{x+0}+\sqrt{x}}$

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§ II

(1.) $V=\frac{1}{3}\pi R^3$ $R=7$, $\frac{dV}{dt}=100$, we WANT $\frac{dR}{dt}$

$\frac{d}{dt}\downarrow$

$$\frac{dV}{dt}=\frac{1}{\cancel{3}}\pi\cdot\cancel{3}R^2\frac{dR}{dt}$$

$$100=\pi(7)^2\cdot\frac{dR}{dt} \Rightarrow \frac{dR}{dt}=\frac{100}{\pi 49}$$

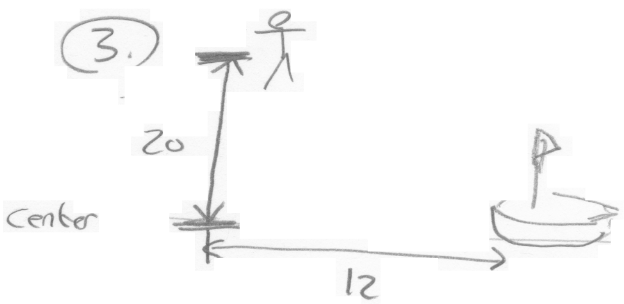
(2.) $V=\frac{1}{3}\pi R^3$ $\frac{dR}{dt}=0.5$, $R=4$ we WANT $\frac{dV}{dt}$

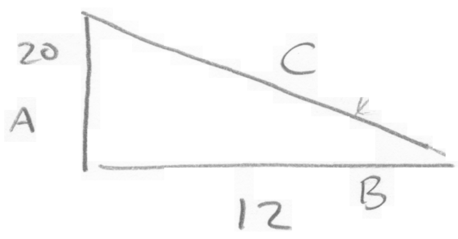
$\frac{d}{dt}\downarrow$

$$\frac{dV}{dt}=\pi R^2\frac{dR}{dt}=\pi(4)^2\cdot(0.5)$$

$$=\boxed{8\pi}$$

(3.)





$20^2+12^2=C^2$

$400+144=C^2$

$\sqrt{544}=C$

$A=20$ $B=12$ $\frac{dB}{dt}=3$

WANT $\frac{dC}{dt}$

USE

$$A^2+B^2=C^2$$

$$20^2+B^2=C^2 \leftarrow \text{sub in CONSTANTS}$$

$$400+B^2=C^2$$

$\frac{d}{dt}\downarrow$

$$2B\frac{dB}{dt}=2C\frac{dC}{dt} \leftarrow \text{SUB IN}$$

$$\cancel{2}(12)\cdot 3=\cancel{2}(\sqrt{544})\frac{dC}{dt} \Rightarrow \frac{dC}{dt}=\frac{36}{\sqrt{544}}=\boxed{\frac{9}{\sqrt{34}}}$$

§ II

4) $PV = nRT$

$$\frac{dP}{dt} \cdot V + P \cdot \frac{dV}{dt} = nR\,\frac{dT}{dt}$$

$$\frac{dP}{dt}\,10 + 7(2) = n \cdot R(-2) \Rightarrow \boxed{\frac{dP}{dt} = \frac{14 - 2nR}{10}}$$

5.)

$$F = G\,\frac{m_1 m_2}{R^2}$$

$$F = G\,m_1 \cdot m_2\,R^{-2}$$

$$\frac{dF}{dt} = G\,m_1 \cdot m_2\left(-2R^{-3}\,\frac{dR}{dt}\right)$$

$$\frac{dF}{dt} = -2G\,m_1 \cdot m_2\,(10)^{-3}(2) = \boxed{\frac{-4G\,m_1 \cdot m_2}{1000}}$$

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§ III

1.) $f(x) = (x+1)^2 (x-1)^3$

$f'(x) = 2(x+1)(x-1)^3 + (x+1)^2 \cdot 3(x-1)^2$

$= (x+1)(x-1)^2 \cdot [2(x-1) + 3(x+1)]$

$= (x+1)(x-1)^2 [2x-2 + 3x+3]$

$f'(x) = (x+1)(x-1)^2 [5x+1] = 0$

$\boxed{x=-1 \quad | \quad x=1 \quad | \quad x=-\frac{1}{5}}$

3 CRITICAL POINTS

++++ $\quad$ – – – – – $\quad$ + + + + $\quad$ + + + +

$-1 \qquad -\frac{1}{5} \qquad 1$

max at $x = -1$

min at $x = -\frac{1}{5}$

$X=1$ NEITHER MAX NOR MIN

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§ III

② $f(x) = e^{x^3-x}$

$$f'(x) = e^{x^3-x} \cdot (3x^2-1) = 0$$

$$e^{x^3-x}(3x^2-1) = 0$$

$$3x^2-1 = 0$$

$$3x^2 = 1 \qquad x^2 = \frac{1}{3} \qquad x = \pm\frac{1}{\sqrt{3}}$$

$+++ \quad - - - \quad +++$

$-\frac{1}{\sqrt{3}} \qquad \frac{1}{\sqrt{3}}$

AT $x = \frac{-1}{\sqrt{3}}$ $f(x)$ HAS A MAX

$x = \frac{+1}{\sqrt{3}}$ $f(x)$ HAS A MIN.

§6

1. $\int \frac{1}{x}\,dx = \ln|x| + C$

2. $\int x^2 - \frac{1}{x} + e^x\,dx = \frac{x^{2+1}}{2+1} - \ln|x| + e^x + C$

Just use Rule,

3. $\int \frac{1}{1+x^2}\,dx = \tan^{-1}(x) + C$

Now we need u-sub

4. $\int e^{3x+1}\,dx$

$$u = 3x+1$$
$$\frac{du}{dx} = 3$$
$$du = 3dx$$
$$\frac{1}{3}du = dx$$

$= \int e^u \frac{1}{3}du$

$= \frac{1}{3}\int e^u du$

$= \frac{1}{3}e^u + C = \boxed{\frac{1}{3}e^{3x+1} + C}$

5. $\int x\, e^{3x^2+1}\,dx$

$$u = 3x^2+1$$
$$du = 6x\,dx$$
$$\frac{1}{6}du = x\,dx$$

$= \int e^{3x^2+1}\, x\,dx$

$= \int e^u \frac{1}{6}du$

$= \frac{1}{6}\int e^u du = \frac{1}{6}e^u + C = \boxed{\frac{1}{6}e^{3x^2+1} + C}$

(6.) $\displaystyle\int \frac{x}{x^2+1}\,dx$ $\qquad u = x^2+1$

$\dfrac{du}{dx} = 2x$

$\displaystyle = \int \frac{\frac{1}{6}du}{u}$ $\qquad du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$\displaystyle = \frac{1}{2}\int \frac{du}{u}$

$\displaystyle = \frac{1}{2}\ln|u| + C = \boxed{\frac{1}{2}\ln|x^2+1| + C}$

(7.) $\displaystyle\int x\sqrt{x^2+1}\,dx$ $\qquad u = x^2+1$

$du = 2x\,dx$

$\displaystyle = \int (x^2+1)^{1/2}\, x\,dx$ $\qquad \frac{1}{2}du = x\,dx$

$\displaystyle = \int (u)^{1/2}\,\frac{1}{2}du$

$\displaystyle = \frac{1}{2}\int u^{1/2}\,du = \frac{1}{2}\,\frac{u^{3/2}}{3/2} + C = \frac{1}{2}\,\frac{)^{3/2}}{} + C$

8.) $\int x \sin(x^2+1)\,dx$

$u = x^2+1$

$\frac{du}{dx} = 2x$

$= \int \sin(x^2+1)\, x\,dx$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$= \int \sin(u)\left(\frac{1}{2}du\right)$

$= \frac{1}{2}\int \sin(u)\,du = \frac{-1}{2}\cos(u) + C = \boxed{-\frac{1}{2}\cos(x^2+1) + C}$

9.) $\int x \sec^2(x^2+1)\,dx$

$u = x^2+1$

$\frac{1}{2}du = x\,dx$

$= \frac{1}{2}\int \sec^2(u)\,du$

$= \frac{1}{2}\tan(u) + C$

This problem done a bit quicker

$= \frac{1}{2}\tan(x^2+1) + C$