QUIZ 5 MA 2310 — ANSWER)

① $y = \dfrac{3x-1}{5x+7}$. COMPUTE $\dfrac{dy}{dx}$ AND $\dfrac{d^2y}{dx^2}$.

$$\frac{dy}{dx} = \frac{3(5x+7) - (3x-1)\,5}{(5x+7)^2} = \frac{(15x+21) - (15x-5)}{(5x+7)^2}$$

$$= \frac{26}{(5x+7)^2}$$

$$\frac{d^2y}{dx^2} = \frac{0\cdot(5x+7)^2 - 26\cdot 2(5x+7)^1\cdot(5)}{[(5x+7)^2]^2} = \boxed{\frac{-260}{(5x+7)^3}}$$

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② COMPUTE THE TANGENT LINE FOR $f(x) = x^2\cdot\sec(x)$

AT $x = \pi$

SOLN: POINT $x_0 = \pi$ TO FIND y_0, PLUG π INTO $f(x)$

SO $y_0 = f(\pi) = (\pi)^2\cdot\sec(\pi) = \pi^2(-1) = -\pi^2 \Rightarrow P = (\pi, -\pi^2)$

SLOPE FIND $f'(x) = 2x\cdot\sec(x) + x^2\cdot\sec(x)\tan(x)$

SO $m = f'(\pi) = 2(\pi)\sec(\pi) + \pi^2\sec(\pi)\tan(\pi)$

$= 2\pi(-1) + \pi^2(-1)\cdot(0)$

$= -2\pi$

SO $y - y_0 = m(x - x_0)$

$$\boxed{y - (-\pi^2) = -2\pi(x - \pi)}$$

(3.)

$S(t) = -16t^2 + 64t + 32$

(a) FIND velocity after t seconds

SOLN: $V(t) = S'(t) = -32t + 64$

(b) At what time does the stone reach the peak?

SOLN The stone reaches its peak when the velocity is zero. So

$0 = -32t + 64$

SOLVE FOR $\boxed{t = 2 \text{ sec}}$

(c) What is that height?

$S(2) = -16(2)^2 + 64(2) + 32 = -64 + 128 + 32 = \underline{96 \text{ feet}}$

(d) When does the stone hit the ground?

↑ that is when is $s = 0$

$S(t) = -16t^2 + 64t + 32$

$0 = -16t^2 + 64t + 32$ ↓ $\div 16$

$0 = -t^2 + 4t + 2$

$t = \dfrac{-4 \pm \sqrt{16 - 4(-1)(2)}}{-2} = \dfrac{-4 \pm \sqrt{24}}{-2}$

$= 2 \pm \sqrt{6}$ $\quad 2 - \sqrt{6}$ is NEGATIVE so

$\boxed{2 + \sqrt{6}}$ is the answer

(e) $V(2+\sqrt{6}) = -32(2+\sqrt{6}) + 64$

$= \boxed{-32\sqrt{6}}$ ← what does a negative velocity mean?

(4.) $C(x) = -0.02x^2 + 50x + 100$ $\quad p(x) = 100 - 0.1x$

(a) $P(x) = x \cdot p(x) - C(x)$

$= x(100 - 0.1x) - (-0.02x^2 + 50x + 100)$

$= 100x - 0.1x^2 + 0.02x^2 - 50x - 100$

$\boxed{P(x) = 50x - 0.08x^2 - 100}$

(b) Average Profit $= \frac{P(x)}{x} = \frac{50x - 0.08x^2 - 100}{x}$

Marginal Profit $= P'(x) = 50 - 0.16x$

(c) Let $x = 500$. Find Marginal Profit & Average Profit.

Average profit $= \frac{50(500) - 0.08(500)^2 - 100}{500} = 9.8$

$= \$9.8$ /item

MARGINAL PROFIT $P'(500) = 50 - 0.16(500) = 50 - 80 = -30$

(d) We profited "on average" \$9.80 per item.

If we make one more item our profit will go down \$30.

(5.) $y = \cos^4(7x^3)$

$$\frac{dy}{dx} = 4\left[\cos(7x^3)\right]^3(-\sin(7x^3)) \cdot 21x^2$$

(6.) $y = (3x^3+4x+1)^{1/2}$

$$\frac{dy}{dx} = \underbrace{\frac{1}{2}\left[3x^3+4x+1\right]^{-1/2}}_{F} \cdot \underbrace{(9x^2+4)}_{S}$$

$$\frac{d^2y}{dx^2} = \underbrace{-\frac{1}{4}\left[3x^3+4x+1\right]^{-3/2} \cdot (9x^2+4)}_{F'} \underbrace{(9x^2+4)}_{S}$$

$$+ \underbrace{\frac{1}{2}\left[3x^3+4x+1\right]^{-1/2}}_{F} \cdot \underbrace{(18x)}_{S'}$$