

Some  
Snowflakes

Frank  
Sanacory

What is  
mathematics?

Dimension

The Snowflake

# Some Snowflakes and Coffee with the Math Department

Frank Sanacory

April 23, 2015

# What is mathematics?

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# What is mathematics?

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- The science of quantity. - Aristotle

# What is mathematics?

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- The science of quantity. - Aristotle
- A mathematician, like a painter or poet, is a maker of patterns. If his patterns are more permanent than theirs, it is because they are made with ideas. - G. H. Hardy, 1940

# What is mathematics?

- Wasn't it all finished long ago? How many new numbers can you invent anyway? Is today's mathematics just a matter of huge calculations, with the mathematician as a kind of zookeeper, making sure the precious computers are fed and watered? If it's not, what is it other than **the incomprehensible outpourings of superpowered brainboxes with their heads in the clouds and their feet dangling from the lofty balconies of their ivory towers?** Mathematics is all of these, and none. Mostly, it's just different. It's not what you expect it to be, you turn your back for a moment and it's changed. It's certainly not just a fixed body of knowledge, its growth is not confined to inventing new numbers, and its hidden tendrils pervade every aspect of modern life. - Ian Stewart (1987) in From Here to Infinity

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# What is mathematics?

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- I shall not today attempt further to define ... But I know it when I see it - Potter Stewart (1964)

# What is mathematics?

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- I shall not today attempt further to define ... But I know it when I see it - Potter Stewart (1964)
- ..it is not philosophy but active experience in mathematics itself that can answer the question: What is mathematics?  
- Richard Courant (in his book What is mathematics?)

# What is mathematics?

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So let's begin...



# Some Dimensional Definitions

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$$\dim(x) = \liminf_n \frac{K(x \upharpoonright n)}{n}$$

$$\text{Dim}(x) = \liminf_n \frac{K(x \upharpoonright n)}{n}$$

# What Dr. More said

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The **dimension** of a commutative ring is the supremum of the lengths of all chains of prime ideals.

# What we will use today

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Let  $X$  be a metric space. If  $S \subseteq X$  and  $d \in [0, \infty)$ , the  **$d$ -dimensional Hausdorff content** of  $S$  is defined by:

$$C_H^d(S) := \inf \left\{ \sum_i r_i^d : \exists \text{ a cover of } S \text{ by balls with radii } r_i > 0 \right\}.$$

The **Hausdorff dimension** of  $X$  is defined by:

$$\dim_H(X) := \inf \{ d \geq 0 : C_H^d(X) = 0 \}.$$

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**the incomprehensible outpourings of superpowered  
brainboxes with their heads in the clouds and their feet  
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# Let's Build the snowflake

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# Let's Build the snowflake

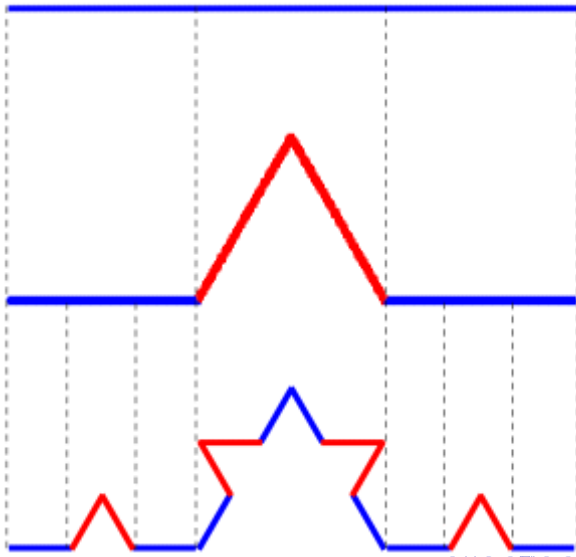
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# So we get something like this

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So we get something like this

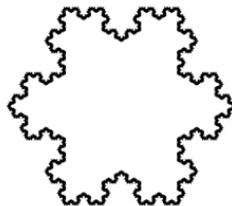
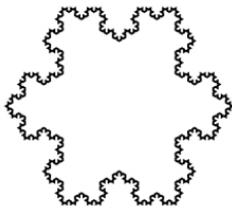
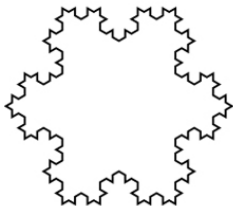
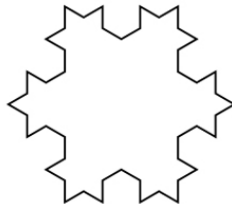
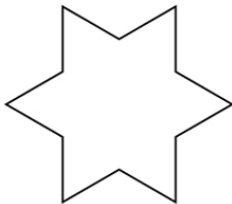
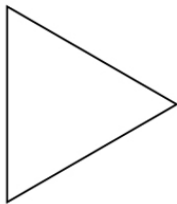
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# Properties of the Koch Snowflake

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## 1 Nowhere differentiable

# Properties of the Koch Snowflake

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- 1 Nowhere differentiable
- 2 Has a very long perimeter . .

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- 1 Nowhere differentiable
- 2 Has a very long perimeter . .
- 3 there is a tessellation of the plane

# Properties of the Koch Snowflake

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- 1 Nowhere differentiable
- 2 Has a very long perimeter . .
- 3 there is a tessellation of the plane
- 4 self similarity

# A Tessellation of the plane

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# A Tessellation of the plane

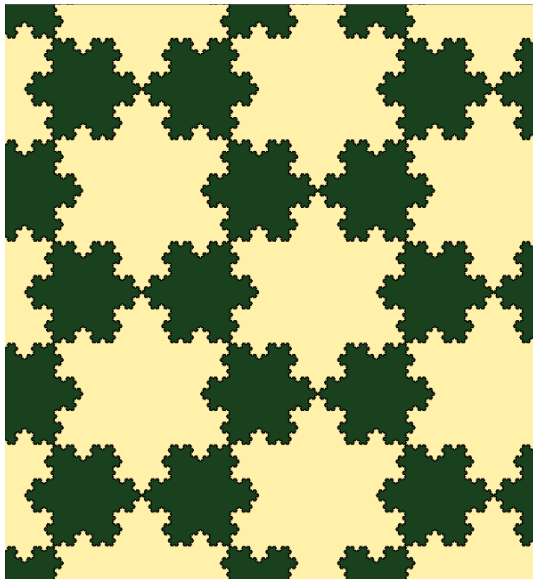
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# Self Similarity

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# Self Similarity

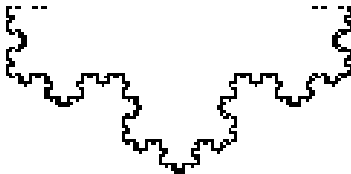
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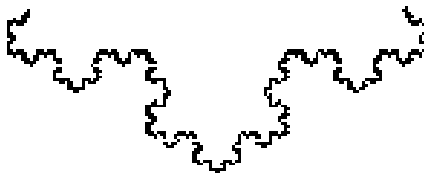
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# More on Self Similarity

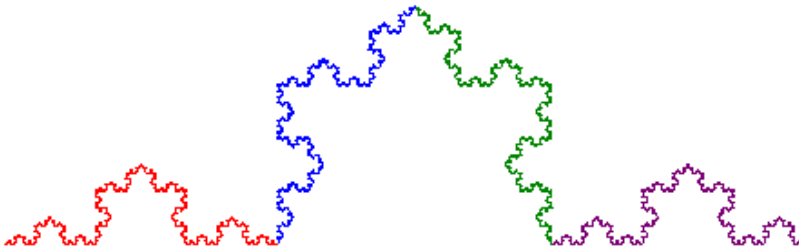
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# And a bit More on Self Similarity

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